unit 1:Introduction to simulation

1. Simulation:

- **Simulation is the imitation of the operation of a real world process or system over time.**
- **Simulation models help us to study the behavior of system as it evolves**
- **models keeps the set of assumption concerning the operation of the system**
- **Assumptions are expressed in terms of mathematical, logical and symbolic relationship between the entities or object of interest of the system.**
- **Simulation modeling can be used both as an analysis tools to predict the performance of the new system and also predict the effect of changes to existing system.**
- **simulation can be done by hand or computer its keeps the history of system**
- **Simulation produce the set of data is used to estimate the measures of performance of system.**

1.1 When Simulation is the Appropriate Tool:

- **Study of and experimentation** with the internal interactions of a complex system, or of a subsystem within a complex system.
- Informational, organizational and environmental changes can be simulated and **the model's behavior can be observer.**
- The knowledge gained in designing a simulation model can be of great **value toward suggesting improvement in the system under investigation.**
- By changing simulation inputs and observing the resulting outputs, valuable insight may be obtained into which variables are most important and how variables interact.
- Simulation can be used as a **pedagogical (teaching) device** to reinforce analytic solution methodologies.
- Can be used to experiment **with new designs or policies prior to implementation,** so as to prepare for what may happen.
- Can be used to **verify analytic solutions.**
- By simulating different capabilities for a machine, requirements can be determined.
- Simulation models **designed for training, allow learning without the cost and disruption of on-the-job instructions.**
- **Animation** shows a system in simulated operation so that **the plan can be visualized**.
- **The modern system (factory, water fabrication plant, service organization, etc) is so complex** that the interactions can be treated only through simulation

1.2 When Simulation is Not Appropriate

õ Simulation should **not be used when the problem can be solved using common sense.**
õ Simulation should **not be used** if the problem can be **solved analytically.**
õ Simulation should **not be used** if it is easier to perform **direct experiments.**
õ Simulation should **not be used**, if the **costs exceeds** savings.
õ Simulation should **not be used** if the **resources or time are not available.**
õ No data is available, not even estimate simulation is not advised.

- If there is not enough time or the people are not available, simulation is not appropriate.
- If managers have unreasonable expectation say, too much soon – or the power of simulation is over estimated, simulation may not be appropriate.
- **If system behavior is too complex or cannot be defined,** simulation is not appropriate

1.3Advantages of Simulation

1. **New policies, operating procedures, decision rules, information flow, etc can be explored** without disrupting the ongoing operations of the real system.
2. **New hardware designs, physical layouts, transportation systems** can be tested without committing resources for their acquisition.
3. **Hypotheses** about how or why certain phenomena occur can be **tested for feasibility.**
4. **Time can be compressed or expanded allowing for a speedup or slowdown** of the phenomena under investigation.
5. Insight can be obtained about the **interaction of variables.**
6. Insight can be obtained about **the importance of variables to the performance of the system.**
7. **Bottleneck analysis** can be performed indication where work-in process, information materials and so on are being excessively delayed.
8. A **simulation study can help in understanding how the system operates** rather than how individuals think the system operates.
9. "what-if" questions can be answered. **Useful in the design of new systems.**

1.4Disadvantages of simulation

1. Model building **requires special training**. It is an art **that is learned over time and through experience.**
2. If two models are **constructed by two competent individuals,** they may have similarities, but it is highly unlikely that they will be the same.
3. Simulation results may be **difficult to interpret.** Since most simulation outputs are essentially random variables (they are usually based on random inputs), it may be hard to determine whether an observation is a result of system interrelationships or randomness.
4. Simulation modeling and analysis can be **time consuming and expensive**. Skimping on resources for modeling and analysis may result in a simulation model or analysis that is not sufficient for the task.
5. Simulation is used in some cases when an analytical solution is possible, or even preferable. This might be particularly true in the simulation of some waiting lines where closed-form queueing models are available.

1.5Applications of Simulation

- **Manufacturing application**
- Semiconductor manufacturing
- construction engineering
- military application
- Business process simulation
- Human system

1. **Manufacturing Applications**
    - Analysis of electronics assembly operations
    - Design and evaluation of a selective assembly station for high-precision scroll compressor shells
    - Comparison of dispatching rules for semiconductor manufacturing using large-facility models
    - Evaluation of cluster tool throughput for thin-film head production
    - Determining optimal lot size for a semiconductor back-end factory
    - Optimization of cycle time and utilization in semiconductor test manufacturing
    - Analysis of storage and retrieval strategies in a warehouse
    - Investigation of dynamics in a service-oriented supply chain
    - Model for an Army chemical munitions disposal facility
2. **Semiconductor Manufacturing**
    - Comparison of dispatching rules using large-facility models
    - The corrupting influence of variability
    - A new lot-release rule for wafer fabs
    - Assessment of potential gains in productivity due to proactive retile management
    - Comparison of a 200-mm and 300-mm X-ray lithography cell
    - Capacity planning with time constraints between operations
    - 300-mm logistic system risk reduction
3. **Construction Engineering**
    - Construction of a dam embankment
    - Trenchless renewal of underground urban infrastructures
    - Activity scheduling in a dynamic, multi project setting
    - Investigation of the structural steel erection process
    - Special-purpose template for utility tunnel construction
4. **Military Application**
    - Modeling leadership effects and recruit type in an Army recruiting station
    - Design and test of an intelligent controller for autonomous underwater vehicles
    - Modeling military requirements for non war fighting operations
    - Using adaptive agent in U.S Air Force pilot retention
5. **Logistics, Transportation, and Distribution Applications**
    - Evaluating the potential benefits of a rail-traffic planning algorithm
    - Evaluating strategies to improve railroad performance
    - Parametric modeling in rail-capacity planning
    - Analysis of passenger flows in an airport terminal
    - Proactive flight-schedule evaluation
    - Logistics issues in autonomous food production systems for extended-duration space exploration
    - Sizing industrial rail-car fleets
    - Product distribution in the newspaper industry
    - Design of a toll plaza

- Choosing between rental-car locations
- Quick-response replenishment

6. **Business Process Simulation**
   - Impact of connection bank redesign on airport gate assignment
   - Product development program planning
   - Reconciliation of business and systems modeling
   - Personnel forecasting and strategic workforce planning
7. **Human Systems and Healthcare**
   - Modeling human performance in complex systems
   - Studying the human element in air traffic control
   - Modeling front office and patient care in ambulatory health care practices
   - Evaluating hospital operations b/n the emergency department and a medical
   - Estimating maximum capacity in an emergency room and reducing length of stay in that room.

1.6 Systems and System Environment

System:

System is defined as a group of object that are joined together in some regular interaction or interdependence toward the accomplishment of same.

System environment:

A system is often affected by changes occurring outside the system,Such changes are said to occure in the system environment.

1.7 Components of a System

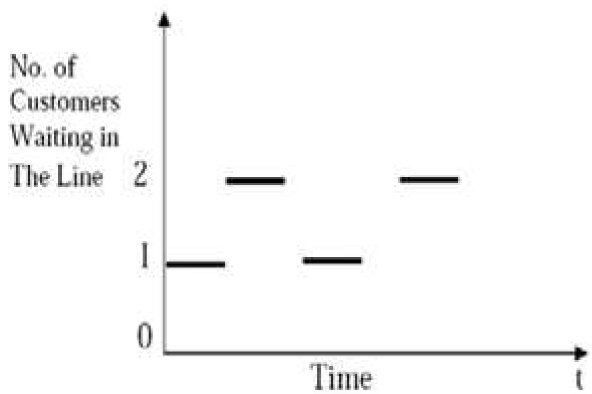
1) **Entity**: An entity is an object of interest in a system.
   Ex: In the factory system, departments, orders, parts and products are the entities.
2) **Attribute**: An attribute denotes the property of an entity.
   Ex: Quantities for each order, type of part, or number of machines in a department are attributes of factory system.
3) **Activity**: Represent a time period of specified length
   Ex: Manufacturing process of the department.
4) **State of the System**: The state of a system is defined as the collection of variables necessary to describe a system at any time, relative to the objective of study.
5) **Event**: An event is defined as an instantaneous occurrence that may change the state of the system.
   Endogenous : IS used to descried activites and events occurring with in the system
   Exogenous: Is used to descried activites and events in the environment that affect the system.

Examples of system and components					
System	**Entities**	**Attributes**	**Activities**	**Events**	**State variables**
Banking	Customers	Checking-account balance	Making deposits	Arrival; departure	No. of busy tellers; no. of customers waiting.
Rapid rail	Riders	Origination; destination	Traveling	Arrival at station; arrival at destination	No. of riders waiting at each station; No. of riders in transit
Production	Machines	Speed; capacity; breakdown rate length	Welding; stamping	Breakdown	Status of machines (busy, idle or down)
Inventory	Warehouse	Capacity	Withdrawing	Demand	Levels of inventory; backlogged demands

1.8 Discrete and Continuous Systems

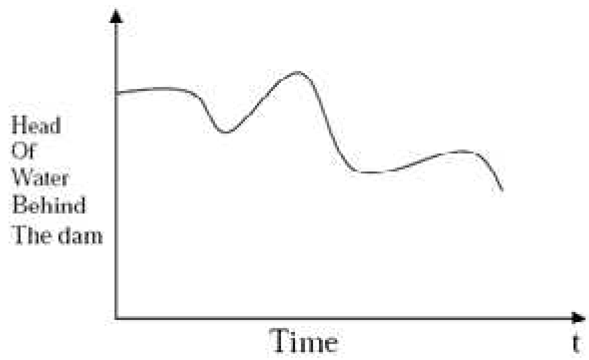
Discrete System:

- **Is one in which the state variable change only at a discrete set of points in time.**
- **The bank is an example, since the state variable the number of customer in the bank changes only when a customer arrives or when the service provided a customer is completed.**



Continuous system:

- **Is one in which the state variable change continuous over time.**
- **head of water behind a dam, during and for some time after a rain storm water flow into the lake behind the dam.**



1.9 Model of a system

- A **model** is defined as a representation of a system for the purpose of studying the system.
- It is necessary to consider only those aspects of the system that affect the problem under investigation.
- These aspects are represented in a model, and by definition it is a simplification of the system.

Types of Models:

- **Mathematical or physical model**
- **Static and dynamic model**
- **deterministic and stochastic model**
- **discrete and continuous model**

1.Mathematical or physical model:

Mathematical model uses symbolic notation and equations to represents a system

2.Static model:

A static simulation models represent a system at a particular point in time it is also called as monte carlo simulation.

3.dynamic model:

A dynamic simulation models represent system as the change over time. simulation of a bank from 9 to 4 is an example

4.Deterministic model:

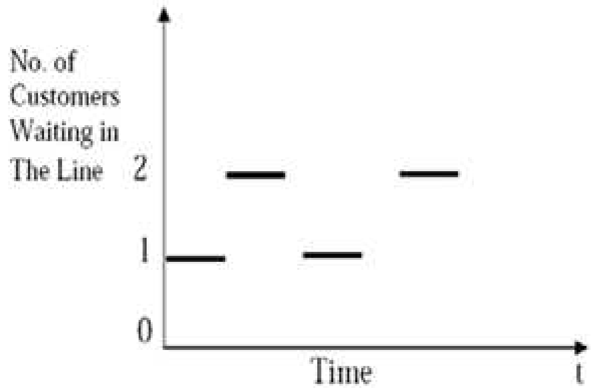
A simulation variable that contain no random variable, have a set of known input which will result in a unique set of output.

5.Stochastic model:

A stochastic simulation model has one or more random **variable** as input. Random input lead to random output.Since the output are random they can be consider only as estimates of the true characteristics of a model.

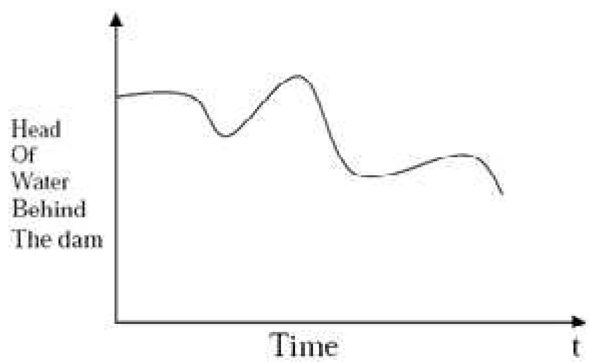
6.Discrete System:

- Is one in which the state variable change only at a discrete set of points in time.
- The bank is an example, since the state variable the number of customer in the bank changes only when a customer arrives or when the service provided a customer is completed.



7.Continuous system:

- Is one in which the state variable change continuous over time.
- head of water behind a dam, during and for some time after a rain storm water flow into the lake behind the dam.



1.10 Discrete event system simulation:

- The model of system in which state variable changes only at a discrete set of points in times
- The simulation models are analyzed by numerical rather than by analytical methods.
- Analytical methods employ the deductive reasoning of mathematics to solve the model. E.g.: Differential calculus can be used to determine the minimum cost policy for some inventory models.
- Numerical methods use computational procedures and are ‘runs’, which is generated based on the model assumptions and observations are collected to be analyzed and to estimate the true system performance measures.
- Real-world simulation is so vast, whose runs are conducted with the help of computer. Much insight can be obtained by simulation manually which is applicable for small

systems.

1.11Steps in a simulation study:

1. Problem formulation
2. Setting of objectives and overall project plan
3. model conceptualization
4. data Collection
5. model translation
6. verified
7. validated
8. Experimental design
9. production runs and analysis
10. more runs
11. documentation and reporting
12. Implementation

1. Problem formulation:

- Every study should begin with a statement of the problem.
- If the statement is provided by the policy makers or those that have the problem, The analyst must ensure that the problem being described is clearly understood
- If the problem statement is being developed by the analyst, it is important that the policy makers understand and agree with the formulation.

2. Setting of objective and overall project plan:

- The objectives indicate the questions to be answered by simulation.
- At this point a determination should be made concerning whether simulation is the appropriate methodology. Assuming that it is appropriate,
- the overall project plan should include the study in terms of
    - A statement of the alternative systems
    - A method for evaluating the effectiveness of these alternatives
    - Plans for the study in terms of the number of people involved
    - Cost of the study
    - The number of days required to accomplish each phase of the work with the anticipated results.

3. Model Conceptualization:

- The construction of a model of a system is probably as much art as science.
- The art of modeling is enhanced by ability to have following:
    - To abstract the essential features of a problem.
    - To select and modify basic assumptions that characterizes the system.
    - To enrich and elaborate the model until a useful approximation results.

4. Data Collection:

- There is a constant interplay between the construction of the model and the collection of the needed input data.
- As complexity of the model changes the required data elements may also change.
- Since data collection takes such a large portion of the total time required to perform a simulation it is necessary to begin it as early as possible.

5. Model Translation:

- Since most real world system result in model that require a great deal of information storage and computation, the model must be entered into a computer recognizable format.
- we use term program even though it is possible to accomplish the desired result in many instances with little or no actual coding.

6.Varified:

- It pertains to the computer program and checking the performance.
- If the input parameters and logical structure and correctly represented, verification is completed.

7.Validated:

- validation is the determination that a model is an accurate representation of the real system.
- Is usually achieved through the calibration of the model an iterative process of comparing the model to actual system behavior and using the discrepancy between the two and the insights gained to improve the model.
- This process is repeated until model accuracy is judges acceptable.

8.**Experimental Design:**

- The alternatives that are to be simulated must be determined. For each system design, decisions need to be made concerning

  a. Length of the initialization period
  b. Length of simulation runs
  c. Number of replication to be made of each run

9.Production runs and analysis:

- They are used to estimate measures of performance for the system designs that are being simulated.

10.More runs:

- Based on the analysis of runs that have been completed. The analyst determines if additional runs are needed and what design those additional experiments should follow.

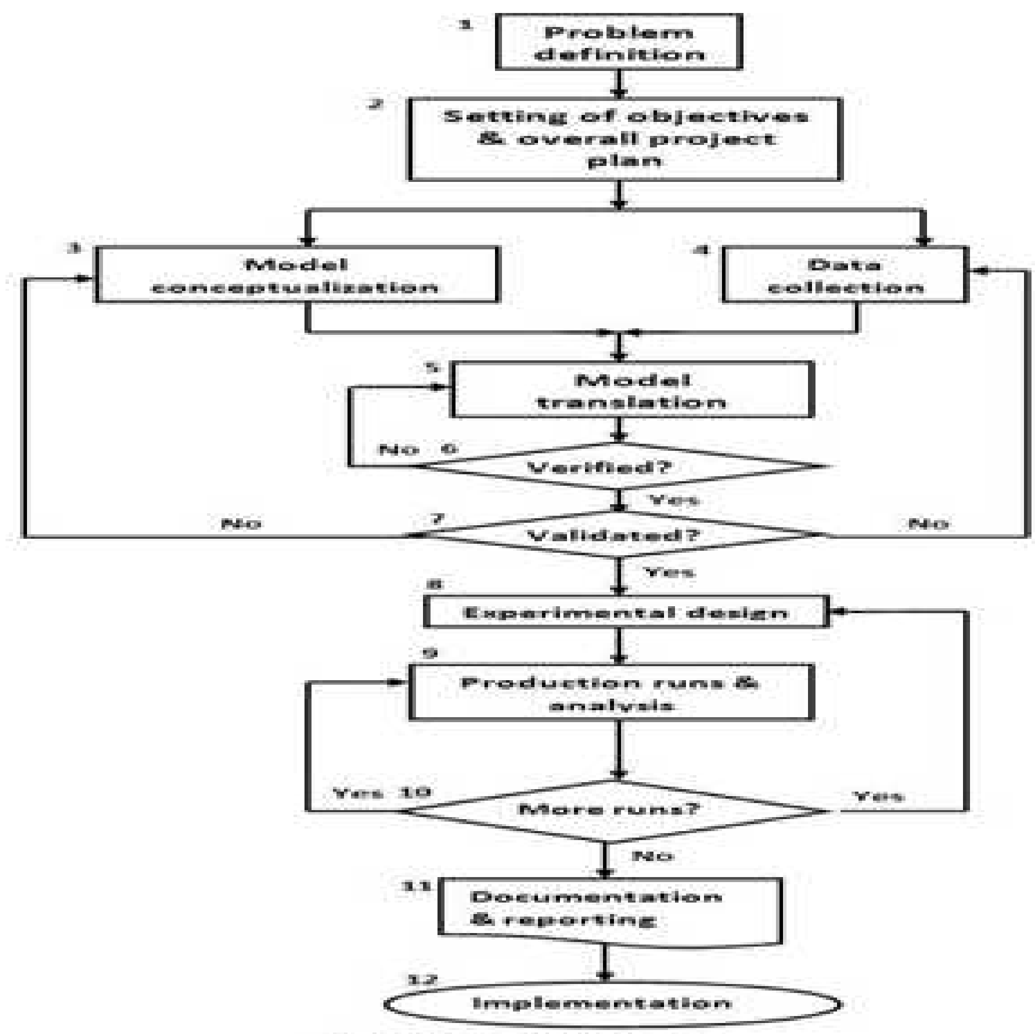
11.Documentation and reporting:

Two types of documentation. Program documentation and Process documentation

- **Program documentation:** Can be used again by the same or different analysts to understand how the program operates
- **Process documentation:** This enable to review the final formulation and alternatives, results of the experiments and the recommended solution to the problem. The final report provides a vehicle of certification.

12.Implementation:

Success depends on the previous steps. If the model user has been thoroughly involved and understands the nature of the model and its outputs, likelihood of a vigorous implementation is enhanced.



Steps in a simulation study

1.12 Simulation of queuing systems

A **Queuing system** is described by **its calling population, the nature of its arrivals, the service mechanism, the system capacity, and queuing discipline.**

Simulation is often used in the analysis of queuing models. In a simple typical queuing model, shown in

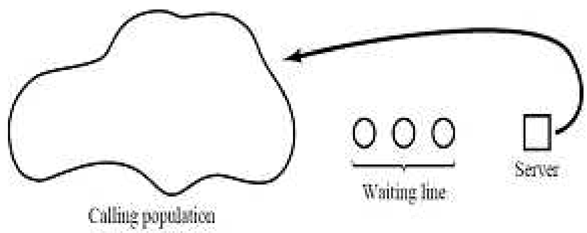


Figure 2 Queueing system.

- In the single-channel queue, the calling population is **infinite**; that is, if a **unit leaves the calling population and joins the waiting line or enters service, there is no change in the arrival rate of other units that may need service.**
- **Arrivals for service occur one at a time in a random fashion**; once they **join the waiting line,** they are eventually served.
- The **system capacity has no limit**, meaning that any number of units can wait in line. Finally, **units are served in the order of their arrival (often called FIFO: first in, first out) by a single server or channel**.
- **Arrivals and services** are defined by the **distributions of the time between arrivals** and the **distribution of service times,** respectively.
- The **state of the system:** the number of units in the system and the status of the server, busy or idle.
- **An event**: a set of circumstances that cause an instantaneous change in the state of the system. In a single-channel queueing system there are only two possible events that can affect the state of the system.
- The **simulation clock** is used to track simulated time.

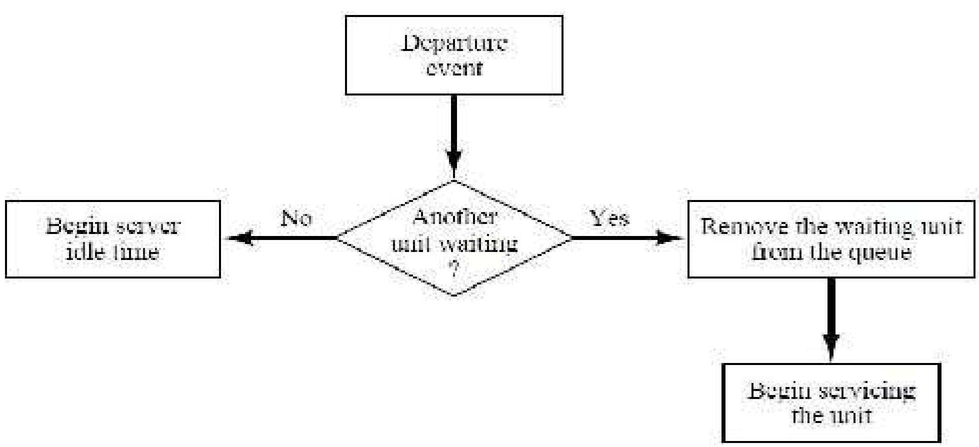


Figure 2.2 Service-just-completed flow diagram.

- The arrival event occurs when a unit enters the system. **The flow diagram for the arrival event is shown in**

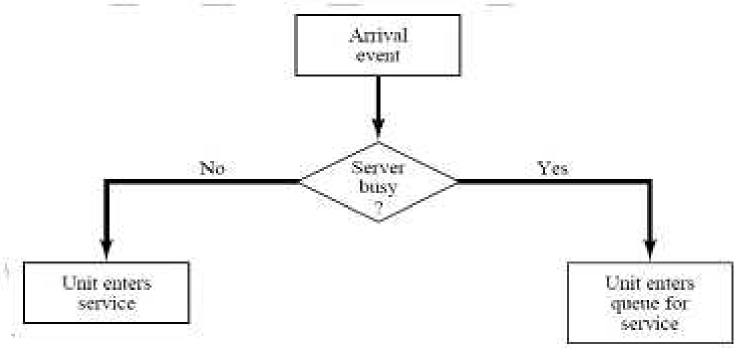


Figure Unit-entering-system flow diagram.

- The unit may find the **server either idle or busy;** therefore, either the unit begins service immediately, or it enters the queue for the server. The unit follows the course of action shown in fig 2.4.
- If the server is busy, the unit enters the queue. If the server is idle and the queue is empty, the unit begins service. It is not possible for the server to be idle and the queue to be nonempty.

		Queue status	
		Not empty	Empty
Server status	Busy	Enter queue	Enter queue
	Idle	Impossible	Enter service

Figure 2.4 Potential unit actions upon arrival.

- After the completion of a service the service may become idle or remain busy with the next unit. The relationship of these two outcomes to the status of the queue is shown in fig 2.5. If the queue is not empty, another unit will enter the server and it will be busy

		Queue status	
		Not empty	Empty
Server outcomes	Busy		Impossible
	Idle	Impossible	

Figure 2.5 Server outcomes after service completion.

Problems:

Single channel queuing system problem formulas:

1. Time Customer wait in queue= Time service begin – Arrival Time
2. Time Service End= Service time + Time service begin
3. Time customer Spend In system= Time service end-Arrival Time
4. Idel Time of Server=Time service Begin(N)-Time Service end(N-1)

Standard Formulas:

1.**Average waiting time(i.e customer wait)**=total time customer wait in queue / Total number of customer

2.**Probability(Wait i.e customer wait)=**Number of Customer who wait / Total number of customer

3.**Probability of idle server(idle time of server)=**total idle time of server / total run time of simulation

4.**average service time**=total service time/total number of customer

5.**average times between arrivals**=sum of all times between arrival/number of arrivals-1

6.**Average waiting time those who wait in queue**=total time customer wait in queue/total number of customer who wait

7.**Average time customer spend In the system**=Total time customer spend in system/total number of customer

① Single channel and Multi channel (Able Baker prob.) Queuing system problem :

UNIT-1

$$\text{Average Waiting time (of customer)} = \frac{\text{Total time customer waiting in queue}}{\text{Total no. of customers}}$$

$$\text{Probability wait (of customer)} = \frac{\text{No. of customers to wait}}{\text{Total no. of customers}}$$

$$\text{Probability of idle server (Idle time of server)} = \frac{\text{Total idle time of server}}{\text{Total run time of simulation (TSE last value)}}$$

$$\text{Average Service Time} = \frac{\text{Total service time}}{\text{Total no. of customers}}$$

$$\text{Average time b/w arrival} = \frac{\text{sum of all time b/w arrival}}{\text{No. of arrival} - 1}$$

$$\text{Average waiting time those who wait in queue} = \frac{\text{Total time customer wait in queu}}{\text{Total no. of customer who wait}}$$

$$\text{Average time customer spends in the system} = \frac{\text{Total time customer spends in S/m}}{\text{Total no. of customers}}$$

AT → Arrival Time

IAT → Inter Arrival Time

ST → Service Time

C.No → Customer Number

RD → Random Digit

RDA → Random Digit Assessment

TSB → Time Service Begin

TSE → Time Service End

CP → Cumulative Probability

P → Probability

1. A small grocery store has only one check out counter. The customer arrives at this check out counter at random from 1 to 8 min apart. Each possible value of service time has same probability of occurance. The service time varies from 1 to 6 mins apart. Each possible value of service time has same probability of occurance. Develop simulation distribution table for 8 customers.

Random digit for arrival time :

913 727 015 948 309 922 753 235 302

Service Time : (Random digit)

84 10 74 53 17 79 91 67 89 38

i) Determine Inter Arrival Time distribution table

C.No	P(1/8)	Cumulative Probability	Random Digit Assessment
1	0.125	0.125	001 – 125
2	0.125	0.250	126 – 250
3	0.125	0.375	251 – 375
4	0.125	0.500	376 – 500
5	0.125	0.625	501 – 625
6	0.125	0.750	626 – 750
7	0.125	0.875	751 – 875
8	0.125	1.000	876 – 000

(2) ii] Compute Arrival time from IAT

C.No	Random Digit	IAT	Arrival Time
1	–	–	0
2	913	8	8
3	727	6	14
4	015	1	15
5	948	8	23
6	309	3	26
7	922	8	34
8	753	7	41
9	235	2	43
10	302	3	46

iii] Determine Service Time Distribution Table

S.No	P (1/6)	Cumulative probability	Random Digit Assess
1	0.16	0.16	01 – 16
2	0.16	0.32	17 – 32
3	0.16	0.48	33 – 48
4	0.16	0.64	49 – 64
5	0.16	0.80	65 – 80
6	0.16	0.96	81 – 96

iv] Compute Service Time

S.No	Random digit	Service Time
1	84	6
2	10	1
3	74	5
4	53	4
5	17	2
6	79	5
7	91	6
8	67	5
9	89	6
10	38	3

v) Simulation Table for 10 customers

					TSE − AT	TSB(n) − TSE(n−1)	TSB − AT
C.No	Arrival Time	Service Time	Time Service Begin	Time Service End	Customer Time spent in System	Idle Server Time	Customer Time wait in queue
1	0	6	0	6	6 (6−0)	0	0
2	8	1	8	9	1 (9−8)	2 (8−6)	0 (8−8)
3.	14	5	14	19	5 (19−14)	5 (14−9)	0 (14−14)
4	15	4	19	23	8	0	4 (19−15)
5	23	2	23	25	2	0	0
6	26	5	26	31	5	1	0
7	34	6	34	40	6	3	0
8	41	5	41	46	5	1	0
9	43	6	46	52	9	0	3
10	46	3	52	55	9	0	6

Average waiting Time $= \dfrac{\text{Total time customer waiting in queue}}{\text{Total no. of customers}} = \dfrac{4+3+6}{10} = \dfrac{13}{10} = \underline{\underline{1.3 \text{ min}}}$

Probability wait $= \dfrac{\text{No. of customers to wait}}{\text{Total no. of customers}} = \dfrac{3}{10} = \underline{\underline{0.3}} \text{ min}$

Probability of idle server $= \dfrac{\text{Total idle time of server}}{\text{Total run time of simulat}^n \text{ (TSE last value)}} = \dfrac{2+5+1+3+1}{55} = \dfrac{12}{55} = \underline{\underline{0.218 \text{ min}}}$

(3) Average Service Time = $\dfrac{\text{Total Service Time}}{\text{Total no. of customers}}$

$$= \frac{6+1+5+4+2+5+6+5+6+3}{10} = \frac{43}{10} = 4.3\text{m}$$

Average Time b/w arrival = $\dfrac{\text{Sum of all time b/w arrival}}{\text{No. of arrival} - 1}$

$$= \frac{46}{10-1} = \frac{46}{9} = 5.11 \text{ min}$$

Average waiting time those who wait in queue = $\dfrac{\text{Total time customer wait in queue}}{\text{Total no. of customer who wait}}$

$$= \frac{4+3+6}{3} = \frac{13}{3} = 4.3 \text{ min}$$

Average time customer spent in system = $\dfrac{\text{Total time customer spends in s/r}}{\text{Total no. of customers}}$

$$= \frac{6+1+5+8+2+5+6+5+9+9}{10}$$

$$= \frac{56}{10} = 5.6 \text{ min}$$

---

2. A small grocery store has only one check out counte at random from 1-6 min apart. Each possible value of IAT has the same probability of occurance. The service time vary from 1 to 6 mins with probability shown below.

ST :	1	2	3	4	5	6
P :	0.10	0.20	0.25	0.30	0.10	0.05

Develop a simulation Table for 10 customers.

Take a random digit for arrival :

13 27 15 48 9 22 53 35 2

Random digit for Service Time

84 10 74 53 17 91 79 67 38 89 sequentially.

Calc. Avg Service Time, probability of idle Service time, average b/w arrivals & avg. time customer spent in System.

i) Determine IAT Distribution Table

C.No	Probability (1/6)	Cumulative probability	Random digit assessment
1	0.16	0.16	01-16
2	0.16	0.32	17-32
3	0.16	0.48	33-48
4	0.16	0.64	49-64
5	0.16	0.80	65-80
6	0.16	0.96	81-96

ii) Compute Arrival time from IAT

C.No	RD	IAT	AT
1	-	-	0
2	13	1	1
3	27	2	3
4	15	1	4
5	48	3	7
6	9	1	8
7	22	2	10
8	53	4	14
9	35	3	17
10	2	1	18

iii) Determine ST Dist. Table

S.No	Probability	CP	RDA
1	0.10	0.10	01-10
2	0.20	0.30	11-30
3	0.25	0.55	31-55
4	0.30	0.85	56-85
		0.95	86-95

iv) Compute Service Time from Distribution Table

S.No	RD	ST
1	84	4
2	10	1
3	74	4
4	53	3
5	17	2
6	91	5
7	79	4
8	67	4
9	38	3
10	89	5

imulation Table for 10 customers

o	Arrival Time	Service Time	Time Service Begin	Time Service End	Cust. time spent in s/m	Idle Server Time	Cust. time wait in queue
	0	4	0	4	4	0	0
	1	1	4	5	4	0	3
	3	4	5	9	6	0	2
	4	3	9	12	8	0	5
	7	2	12	14	7	0	5
	8	5	14	19	11	0	6
	10	4	19	23	13	0	9
	14	4	23	27	13	0	9
	17	3	27	30	13	0	10
	18	5	30	35	17	0	12

erage Service Time $= \frac{35}{10} = \underline{3.5 \text{ min}}$

bability of Idle Server Time $= \frac{0}{35} = \underline{0}$

erage Time b/w arrivals $= \frac{18}{9} = \underline{2 \text{ min}}$

erage time Customer spent in system $= \frac{61}{10} = \underline{6.1 \text{ min}}$

3. Consider a store with one checkout counter. Prepare simulation table & find out average waiting time of customer in waiting queue, probability of idle server, average service time

*

IAT : 3 2 6 4 4 5 8 7

ST : 4 5 5 8 4 6 2 3 4

Assume 1^{st} customer arrives at $t=0$.

i] Inter arrival distribution Table

C.No	IAT	AT
1	–	0
2	3	3
3	2	5
4	6	11
5	4	15
6	4	19
7	5	24
8	8	32
9	7	39

ii] Simulation table for 9 customers

C.No	AT	ST	TSB	TSE	Cust. spent in s/m.	Idle time of server	Cust. time wait in que
1	0	4	0	4	4	0	0
2	3	5	4	9	6	0	1
3	5	5	9	14	9	0	4
4	11	8	14	22	11	0	3
5	15	4	22	26	11	0	7
6	19	6	26	32	13	0	7
7	24	2	32	34	10	0	8
8	32	3	34	37	5	0	2
9	39	4	39	43	4	2	0

(5) Average waiting time of customer in waiting queue $= \frac{32}{7} = 4.57$ min

Probability of Idle Server $= \frac{2}{10} = 0.2$ min

Average Service Time $= \frac{41}{9} = 4.56$ min

Multichannel Problems :

Type I

4 Consider a simulation with a restaurant system where car ~~&~~ hope takes order & brings an item to the car. The car arrives in the manner :

Time b/w arrival :	1	2	3	4
Probability :	0.25	0.40	0.20	0.15

Consider 2 persons Able & Baker. Able is better & bit faster than Baker.

Able Service Time :

ST :	2	3	4	5
Prob :	0.30	0.28	0.25	0.17

Baker Service Time :

ST :	3	4	5	6
Prob :	0.35	0.25	0.20	0.20

Take a random digit for arrival :

26 98 90 26 42 74 80 68 22

Random digit for service time :

95 21 51 92 89 38 13 61 50 49

i) Determine Inter Arrival distribution Table

C.No	Probability	Cumulative Probability	Random Digit Assessment
1	0.25	0.25	01-25
2	0.40	0.65	26-65
3	0.20	0.85	66-85
4	0.15	1.00	86-00

ii) Compute AT from IAT distribution Table

C.No	RD	IAT	AT
1	-	-	0
2	26	2	2
3	98	4	6
4	90	4	10
5	26	2	12
6	42	2	14
7	74	3	17
8	80	3	20
9	68	3	23
10	22	1	24

iii) Able Service Time Distribution Table

S.No	Probability	CP	RDA
2	0.30	0.30	01-30
3	0.28	0.58	31-58
4	0.25	0.83	59-83
5	0.17	1.00	84-00

iv) Baker Service Time Distribution Table

S.No	Probability	CP	RDA
3	0.35	0.35	01-35
4	0.25	0.60	36-60
5	0.20	0.80	61-80
6	0.20	1.00	81-00

6) v] Simulation table for 10 customers

C.No	AT	RD for service	ST	When Able is Available	When Baker is Available	Server Choosen	Able TSB	Able TSE	Baker TSB	Baker TSE	cust in s/m	Idle Server time	wa in que
1	0	95	5	0	0	A	0	5	–	–	5	0	0
2	2	21	3	5	0	B	–	–	2	5	3	2	0
3	6	51	3	5	5	A	6	9	–	–	3	1	0
4	10	92	5	9	5	A	10	15	–	–	5	1	0
5	12	89	6	15	5	B	–	–	12	18	6	7	0
6	14	38	3	15	18	A	15	18	–	–	4	0	1
7	17	13	2	18	18	A	18	20	–	–	3	0	1
8	20	61	4	20	18	A	20	24	–	–	4	0	0
9	23	50	4	24	18	B	–	–	23	27	4	5	0
10	24	49	3	24	27	B	24	27	–	–	3	0	0

5. Consider Simulation table for Able & Baker problem where time b/w arrival are :

Time b/w arrival : 1 2 3 4 5

Probability : 0.20 0.15 0.05 0.20 0.40

Baker is faster than Able.

Service Time for Able :

ST : 3 4 5 6 2

Prob : 0.20 0.05 0.15 0.20 0.40

Service Time for Baker :

ST : 2 3 5 6 1

Prob : 0.15 0.20 0.05 0.20 0.40

Random Digit for Arrival :

98 90 42 80 22 26 74 26 68

Random Digit for Service Time :

49 50 61 13 38 89 92 51 21 95

i) Determine IAT distribution Table

C.No	Probability	CP	RDA
1	0.20	0.20	01-20
2	0.15	0.35	21-35
3	0.05	0.40	36-40
4	0.20	0.60	41-60
5	0.40	1.00	61-00

ii) Compute Arrival Time from IAT dist. table.

C.No	RD	IAT	AT
1	-	-	0
2	98	5	5
3	90	5	10
4	42	4	14
5	80	5	19
6	22	2	21
7	26	2	23
8	74	5	28
9	26	2	30
10	68	5	35

iii) Able Service Time Distribution Table

S.No	P	CP	RDA
3	0.20	0.20	01-20
4	0.05	0.25	21-25
5	0.15	0.40	26-40
6	0.20	0.60	41-60
2	0.40	1.00	61-00

iv) Baker Service Time Distribution Table

S.No	P	CP	RDA
2	0.15	0.15	01-15
3	0.20	0.35	16-35
5	0.05	0.40	36-40
6	0.20	0.60	41-60
1	0.40	1.00	61-00

(RD - Random Digit)

v] Simulation table for 10 customers

C.No	Arrival Time	RD of service	Service Time	When Able is Available	When Baker is Available	Server Choosen	Able		Baker		Cust. time spent in s/m	Idle time of Server	Cust. t[…] wait[…] queu[…]
							TSB	TSE	TSB	TSE			
1	0	49	6	0	0	B	—	—	0	6	6	0	0
2	5	50	6	0	6	A	5	11	—	—	6	5	0
3	10	61	1	11	6	B	—	—	10	11	1	4	0
4	14	13	2	11	11	B	—	—	14	16	2	3	0
5	19	38	5	11	16	B	—	—	19	24	5	3	0
6	21	89	2	11	24	A	21	23	—	—	2	10	0
7	23	92	2	23	24	A	23	25	—	—	2	0	0
8	28	51	6	25	24	B	—	—	28	34	6	4	0
9	30	21	4	25	34	B	30	34	—	—	4	5	0
10	35	95	1	34	34	A	—	—	35	36	1	1	0

Customer time Spent in s/m = TSE - AT

Idle time of Server = TSB(n) - TSE(n-1)

Customer time waiting in queue = TSB - AT

6. Consider Able Baker Dist. table, IAT having equal probability ratio 1 to 6 min apart.

* Able Service Time have equal probability ratio 1 - 5 min

Baker Service Time have equal probability ratio 1 - 4 min.
1 - 6

Random Digit for time b/w arrival :

62 89 9 62 24 47 8 86 22

Random Digit for Service Time :

59 12 15 29 28 83 13 16 5 94

Able is faster than Baker.

i) Determine IAT Distribution Table

C.No	P	CP	RDA
1	0.16	0.16	01-16
2	0.16	0.32	17-32
3	0.16	0.48	33-48
4	0.16	0.64	49-64
5	0.16	0.80	65-80
6	0.16	0.96	81-96

ii) Compute Arrival Time from IAT Dist. Table

C.No	RD	IAT	AT
1	-	-	0
2	62	4	4
3	89	6	10
4	9	1	11
5	62	4	15
6	24	2	17
7	47	3	20
8	8	1	21
9	86	6	27
10	22	2	29

· Distribution Table

S·No	P	CP	RDA
1	0·20	0·20	01-20
2	0·20	0·40	21-40
3	0·20	0·60	41-60
4	0·20	0·80	61-80
5	0·20	1·00	81-00

Distribution Table

S·No	P	CP	RDA
1	0·25	0·25	01-25
2	0·25	0·50	26-50
3	0·25	0·75	51-75
4	0·25	1·00	76-00

v) Simulation Table for 10 customers

C·No	AT	RD of service	ST	When Able is Available	When Baker is Available	Server Choosen	Able		Baker		Cust. spent in s/m	Idle time of server	Cust. time wait in queue
							TSB	TSE	TSB	TSE			
1	0	59	3	0	0	A	0	3	–	–	3	0	0
2	4	12	1	3	0	A	4	5	–	–	1	1	0
3	10	15	1	5	0	A	10	11	–	–	1	5	0
4	11	29	2	11	0	A	11	13	–	–	2	0	0
5	15	98	15	13	0	A	15	20	–	–	2	2	0
6	17	83	4	20	0	B	–	–	17	21	2	17	0
7	20	13	1	20	21	A	20	21	–	–	1	0	0
8	21	16	1	21	21	A	21	22	–	–	1	0	0
9	27	5	1	22	21	A	27	28	–	–	2	5	0
10	29	94	5	28	21	A	29	34	–	–	5	1	0

7. Develop a simulation table for Able & Baker problem.

Inter Arrival Times : 3 2 1 5 4 6

Service Time (Able) : 3 5 2 6 2 1 7

Service Time (Baker): 2 1 4 5 7 6 2

Able is much faster than Baker

i) Compute AT from IAT

C. No	IAT	AT
1	–	0
2	3	3
3	2	5
4	1	6
5	5	11
6	4	15
7	6	21

ii) Simulation Table for 7 customers

C.No	AT	ST	When Able is Available	When Baker is Available	Server choosen	Able		Baker		Cust. spent in s/m	Idle time of Server	Cust. time wait in queue
						TSB	TSE	TSB	TSE			
1	0	3	0	0	A	0	3	–	–	0	0	3
2	3	5	3	0	A	3	8	–	–	0	0	5
3	5	4	8	0	B	–	–	5	9	0	5	4
4	6	6	8	9	A	8	14	–	–	2	0	8
5	11	7	14	9	B	–	–	11	18	0	2	7
6	15	1	14	18	A	15	16	–	–	0	1	1
7	21	7	16	18	A	21	28	–	–	0	5	7

(9) having equal probability ratio 1 to 7 min apart.

Service Time for Able :

ST :	1	2	3	4	5
Prob :	0.20	0.10	0.30	0.20	0.20

Service Time for Baker :

ST :	1	2	3	4	5
Prob. :	0.10	0.20	0.20	0.30	0.20

Random Visit for Arrival : 95 60 35 40 52 54 10

Random visit for Service : 60 95 35 40 24 54 10 25

Baker is faster than Able.

i) Determine Inter Arrival Distribution Table.

C.No	P	CP	RDA
1	0.14	0.14	01-14
2	0.14	0.28	15-28
3	0.14	0.42	29-42
4	0.14	0.56	43-56
5	0.14	0.70	57-70
6	0.14	0.84	71-84
7	0.14	0.98	85-98

ii) Compute AT from IADT.

C.No	RD	IAT	AT
1	–	–	0
2	95	7	7
3	60	5	12
4	35	3	15
5	40	3	18
6	52	4	22
7	54	4	26
8	10	1	27

iii) Determine Able Service Time Distribution Table

S.No	P	CP	RDA
1	0.20	0.20	01-20
2	0.10	0.30	21-30
3	0.30	0.60	31-60
4	0.20	0.80	61-80
5	0.20	1.00	81-00

iv) Determine Baker Service Time Distribution table

S.No	P	CP	RDA
1	0.10	0.10	1-10
2	0.20	0.30	11-30
3	0.20	0.50	31-50
4	0.30	0.80	51-80
5	0.20	1.00	81-00

v] Simulation Table for 8 customers.

C.No	Arrival Time	RD of Service	Service Time	When Able is Available	When Baker is Available	Server Choosen	Able		Baker		Cust. spent in s/m	Idle Server Time	Cust·W in que
							TSB	TSE	TSB	TSE			
1	0	60	4	0	0	B	–	–	0	4	4	0	0
2	7	95	5	0	4	B	–	–	7	12	5	3	0
3	12	35	3	0	12	B	–	–	12	15	3	0	0
4	15	40	3	0	15	B	–	–	15	18	3	0	0
5	18	24	2	0	18	B	–	–	18	20	2	0	0
6	22	54	4	0	20	B	–	–	22	26	4	2	0
7	26	10	1	0	26	B	–	–	26	27	1	0	0
8	27	25	2	0	27	B	–	–	27	29	2	0	0

General Principles

1. Discrete-event simulation

- The basic building blocks of all discrete-event simulation models: entities and attributes, activities and events.
- A system is modeled in terms of
  - Its state at each point in time
  - The entities that pass through the system and the entities that represent system resources
  - The activities and events that cause system state to change.
- Discrete-event models are appropriate for those systems for which changes in system state occur only at discrete points in time.
- This chapter deals exclusively with dynamic, stochastic systems (i.e., involving time and containing random elements) which change in a discrete manner.

Concepts in Discrete-Event Simulation(components of discrete event Simulation)

1. **System:** A collection of entities (e.g., people and machines) that together over time to accomplish one or more goals.
2. **Model:** An abstract representation of a system, usually containing structural, logical, or mathematical relationships which describe a system in terms of state, entities and their attributes, sets, processes, events, activities, and delays.
3. **System state:** A collection of variables that contain all the information necessary to describe the system at any time.
4. **Entity:** Any object or component in the system which requires explicit representation in the model (e.g., a server, a customer, a machine).
5. **Attributes:** The properties of a given entity (e.g., the priority of a v customer, the routing of a job through a job shop).
6. **List:** A collection of (permanently or temporarily) associated entities ordered in some logical fashion (such as all customers currently in a waiting line, ordered by first come, first served, or by priority).
7. **Event:** An instantaneous occurrence that changes the state of a system as an arrival of a new customer).
8. **Event notice:** A record of an event to occur at the current or some future time, along with any associated data necessary to execute the event; at a minimum, the record includes the event type and the event time.
9. **Event list:** A list of event notices for future events, ordered by time of occurrence; also known as

the future event list (FEL).

10. **Activity:** A duration of time of specified length (e.g., a service time or arrival time), which is known when it begins (although it may be defined in terms of a statistical distribution).
11. **Delay:** A duration of time of unspecified indefinite length, which is not known until it ends (e.g., a customer's delay in a last-in, first-out waiting line which, when it begins, depends on future arrivals).
12. **Clock:** A variable representing simulated time.

The Event-Scheduling/Time-Advance Algorithm

- **The mechanism for advancing simulation time and guaranteeing that all events occur in correct chronological order is based on the future event list (FEL).**
- **Future Event List (FEL)**
  - **To contain all event notices for events that have been scheduled to occur at a future time.**
  - **To be ordered by event time,** meaning that the events are arranged chronologically; that is, the event times satisfy.
  - **Scheduling a future event** means that at the instant an activity begins, its duration is computed or drawn as a sample from a statistical distribution and the end-activity event, together with its event time, is placed on the future event list.

The sequence of actions which a simulator must perform to advance the clock system snapshot is called the event- scheduling/time-advance algorithm.

The system snapshot at time t=0 and t=t1 (VIP VTU question)

ClK	System State		Future Event List	
T	(5,1,6)		(3, t1)— Type 3 event to occur at timet1 (1, t2)— Type 1 event to occur at time t2 (1, t3)- Type 1 event to occur at time t3 (2, tn)— Type 2 event to occur at time tn	

Event-scheduling/time-advance algorithm

Step 1. Remove the event notice for the imminent event

(event 3, time t\) from FEL

Step 2. Advance CLOCK to imminent event time

(i.e., advance CLOCK from r to t1).

Step 3. Execute imminent event: update system state, change entity attributes, and set membership as needed.

Step 4. Generate future events (if necessary) and place their event notices on PEL ranked by event time.

(Example: Event 4 to occur at time t*, where t2 < t* < t3.)

Step 5. Update cumulative statistics and counters.

New system snapshot at time *t1*

OCK	System		Future Event List	
T1	(5,1,5)		(1, t2)— Type 1 event to occur at time t1 (4, t*)— Type 4 event to occur at time t* (1, t3)— Type 1 event to occur at time t3 (2, tn)—Type 2 event to occur at time tn	

2.Manual Simulation Using Event Scheduling

In an event-scheduling simulation, a simulation table is used to record the successive system snapshots as time advances.

Let us consider the example of a grocery shop which has only one checkout counter. **(Single-Channel Queue)**

The system consists of those customers in the waiting line plus the one (if any) checking out. The model has the following components:

System state (LQ (t), LS (t)), where LQ (t) is the number of customers in the waiting line, and LS (t) is the number being served (0 or 1) at time t.

Entities: The server and customers are not explicitly modeled, except in terms of the state variables above.

Events

Arrival(A)

Departure(D)

Stopping event (E), scheduled to occur at time 60.

Event notices

(A, t). Representing an arrival event to occur at future time t

(D, t), representing a customer departure at future time t

(E, 60), representing the simulation-stop event at future time 60

Activities

Interarrival time, Service time,

Delay Customer time spent in waiting line.

In this model, the FEL will always contain either two or three event notices.

Flow Chart for execution of arrival and departure event using time advance /Event scheduling algorithm (vtu Question)

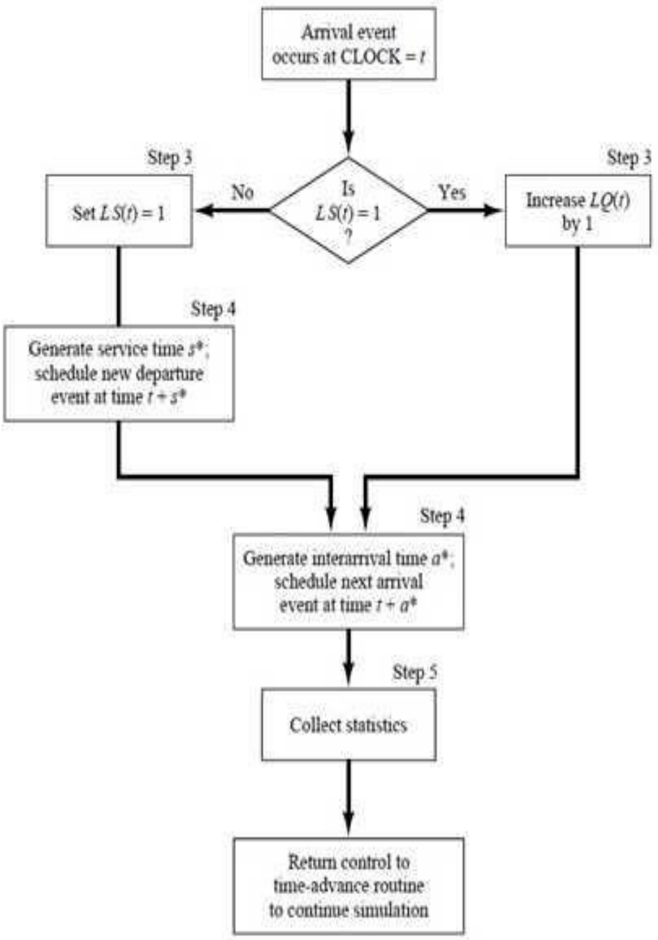


Figure Execution of the arrival event.

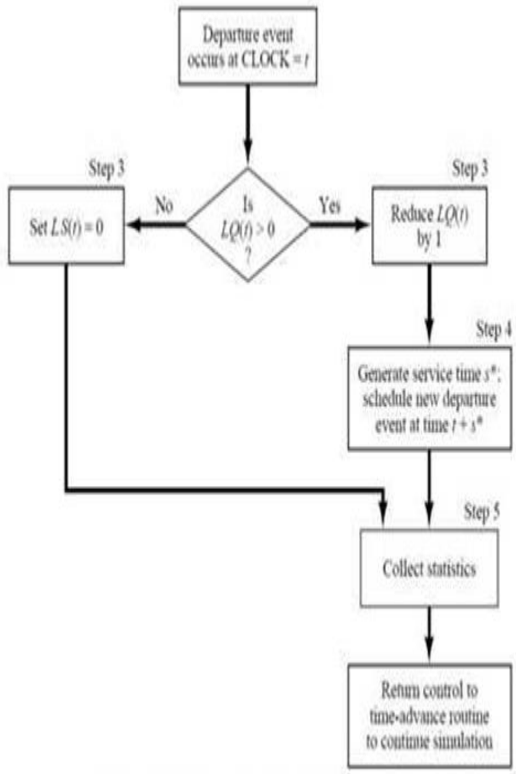


Figure Execution of the departure event.

Question Bank

1. When the simulation is appropriate tool & when it is not.
2. Advantages & disadvantages of simulation.
3. Components of systems & model and it types.
4. Steps in simulation study.
5. Examples (single server channel queue refer 2015, 2014, 2013 question paper, & class problem.
6. Examples Able & Bakes call center problem (two channel server problem)
7. Explain the terms used in discrete event simulation with an example(Ex. Able & Baker)
8. Explain the event scheduling algorithm by generating system snapshots at clock =t and clock=t1.
9. Explain the event scheduling algorithm with an example (single-channel-queue – execution of arrival event & execution of departure event).

UNIT-2 [NOTE: Check out time column should be included in prob. No 1 to 5. It is included in prob. 6 to 8.]

Problems on Event Scheduling / Time advanced Algorithm

1. Prepare a simulation table for 1 channel queuing s/m using ES/TA algorithm. Stopping event is at 30.

IAT : 8 6 1 8 2 8

ST : 4 1 4 3 2 4

LQ → Load Queue

LS → Load Service

Step 1 : Compute Departure Time

C.No	Inter Arrival Time	Arrival Time	Service Time	Departure Time
1	–	0	4	4
2	8	8	1	9 (8+1)
3	6	14	4	18 (14+4)
4	1	15	3	21 (18+3)
5	8	23	2	25 (23+2)
6	2	25	4	29 (25+4)
7	8	33	–	–

Step 2 : Simulation Table for 6 customers

Event Type	Clock	System State		Future Event List	Cumulative Statistics	
		LQ(E)	LS(E)		Busy	Maximum Queue
A_1	0	0	1	(D,4) (A,8) (E,30)	0	0
D_1	4	0	0	(A,8) (D,9) (E,30)	4	0
A_2	8	0	1	(D,9) (A,14) (E,30)	4	0
D_2	9	0	0	(A,14) (D,18) (E,30)	5	0
A_3	14	0	1	(A,15) (D,18) (E,30)	5	0
A_4	15	1	1	(D,18) (A,23) (E,30)	6	1
D_3	18	0	1	(D,21) (A,23) (E,30)	9	1
D_4	21	0	0	(A,23) (D,25) (E,30)	12	1
A_5	23	0	1	(D,25) (A,25) (E,30)	12	1

A_6/D_5	25	0	1	(A, 33) (D, 29) (E, 30)	14	1
D_6	29	0	0	(E, 30) (A, 33)	14	1
~~D_6~~	~~29~~	~~0~~	~~0~~	~~(E, 30)~~	~~18~~	~~4~~

2. Stopping Time = 30

IAT : 2 4 3 1 2 5 6

ST : 4 3 2 5 2 1

i] Compute Departure Time

C. No	IAT	AT	ST	DT
1	–	0	4	4
2	2	2	3	7
3	4	6	2	9
4	3	9	5	14
5	1	10	2	16
6	2	12	1	17
7	5	17	–	–
8	6	23	–	–

ii] Simulation Table for 6 customers

Event Type	Clock	System State		Future Event List	CS	
		LQ(t)	LS(t)		B	MQ
A_1	0	0	1	(A, 2) (D, 4) (E, 30)	0	0
A_2	2	1	1	(D, 4) (A, 6) (E, 30)	2	1
D_1	4	0	1	(A, 6) (D, 7) (E, 30)	4	1
A_3	6	1	1	(D, 7) (A, 9) (E, 30)	6	1
D_2	7	0	1	(A, 9) (D, 9) (E, 30)	7	1
A_4/D_3	9	0	1	(A, 10) (D, 14) (E, 30)	9	1
~~A_4~~	[illegible]	[illegible]	[illegible]	~~(A, 10) (D, 14) (E, 30)~~	[illegible]	[illegible]

(2) A_6	12	2	1	(D, 14) (A, 17) (E, 30)	12	2
D_4	14	1	1	(A, 17) (D, 16) (E, 30)	14	2
D_5	16	1	1	(A, 17) (D, 17) (E, 30)	16	2
A_7/D_6	17	0	1	(A, 23) (E, 30)	17	2

3. Stopping Time = 60

IAT : 1 1 6 3 7 5 2 4 1

ST : 4 2 5 4 1 5 4 1 4

i) Compute Departure Time

C. No	IAT	AT	ST	DT
1	–	0	4	4
2	1	1	2	6
3	1	2	5	11
4	6	8	4	15
5	3	11	1	16
6	7	18	5	23
7	5	23	4	27
8	2	25	1	28
9	4	29	4	33
10	1	–	–	–

ii) Simulation Table for 9 customers

Event Type	Clock	S/m State LQ(t)	S/m State LS(t)	Future Event List	CS B	CS MQ
A_1	0	0	1	(A, 1) (D, 4) (E, 60)	0	0
A_2	1	1	1	(A, 2) (D, 4) (E, 60)	1	1
A_3	2	2	1	(D, 4) (A, 8) (E, 60)	2	2
D_1	4	1	1	(D, 6) (A, 8) (E, 60)	4	2
D_2	6	0	1	(A, 8) (D, 11) (E, 60)	6	2
A_4	8	1	1	(A, 11) (D, 11) (E, 60)	8	2

D_3/A_5	11	1	1	(D,15) (A,18) (E,60)	11	2
~~D_3~~	[illegible]	[illegible]	[illegible]	~~(D,15) (A,18) (E,60)~~	[illegible]	[illegible]
D_4	15	0	1	(D,16) (A,18) (E,60)	15	2
D_5	16	0	0	(A,18) (D,23) (E,60)	16	2
A_6	18	0	1	(D,23) (A,23) (E,60)	16	2
D_6	23	0	1	(A,23) (D,27) (E,60)	21	2
A_7	23	0	1	(D,27) (A,25) (E,60)	21	2
A_8	25	1	1	(D,27) (A,29) (E,60)	23	2
D_7	27	0	1	(A,29) (A,28) (E,60)	25	2
D_8	28	0	0	(A,29) (D,33) (E,60)	26	2
A_9	29	0	1	(D,33) (E,60)	26	2
D_9	33	0	0	(E,60)	30	2

4. Prepare a simulation table using ES/TA algorithm until the clock reaches time 23 using IAT & ST given :
Stopping Time is 30.

IAT : 5 1 2 3 4 9 5 8 6 1

ST : 4 7 8 1 4 2 5 3 1 4

(i) Compute Departure Time

C. No	IAT	AT	ST	DT
1	–	0	4	4
2	5	5	7	12
3	1	6	8	20
4	2	8	1	21
5	3	11	4	25
6	4	15	2	27
7	9	24	5	32
8	5	29	3	35

C. No	IAT	AT	ST	DT
9	8	37	1	38
10	6	43	4	47
11	1	–	–	–

(3) ii) Simulation Table for 9 customers.

Event Type	Clock	S/m state LQ(t)	LS(t)	Future Event List	CS B	MQ
A_1	0	0	1	(A,1) (D,4)	0	0
A_2	1	1	1	(D,4) (A,7)	1	1
D_1	4	0	1	(D,6) (A,7)	4	1
D_2	6	0	0	(A,7) (D,12)	6	1
A_3	7	0	1	(A,8) (D,12)	6	1
A_4	8	1	1	(A,11) (D,12)	7	1
A_5	11	2	1	(D,12) (A,16)	10	2
D_3	12	1	1	(A,16) (D,17)	11	2
A_6	16	2	1	(D,17) (A,23)	15	2
D_4	17	1	1	(D,18) (A,23)	16	2
D_5	18	0	1	(D,22) (A,23)	17	2
D_6	22	0	0	(A,23) (D,27)	21	2
A_7	23	0	1	(A,24) (D,27)	21	2
A_8	24	1	1	(D,27) (A,28)	22	2
D_7	27	0	1	(A,28) (D,31)	25	2
A_9	28	1	1	(D,31) (D,32)	26	2
D_8	31	0	1	(D,32)	29	2
D_9	32	1	0	–	30	2

S → Customer who taken response time from system

N_D → No. of customer departured

F → customer who spent more than given time (Eg: 4 min)

S = Response Time + Current Departure

Response Time = Clock − Current Arrival

ii) Simulation Table

Event Type	Clock	S/m state LQ(t)	LS(t)	Future Event List	CS B	MQ
A_1	0	0	1	(D, 4) (A, 5) (E, 30)	0	0
D_1	4	0	0	(A, 5) (D, 12) (E, 30)	4	0
A_2	5	0	1	(A, 6) (D, 12) (E, 30)	4	0
A_3	6	1	1	(A, 8) (D, 12) (E, 30)	5	1
A_4	8	2	1	(A, 11) (D, 12) (E, 30)	7	2
A_5	11	3	1	(D, 12) (A, 15) (E, 30)	10	3
D_2	12	2	1	(A, 15) (D, 20) (E, 30)	11	3
A_6	15	3	1	(D, 20) (A, 24) (E, 30)	14	3
D_3	20	2	1	(D, 21) (A, 24) (E, 30)	19	3
D_4	21	1	1	(A, 24) (D, 25) (E, 30)	20	3

5. Prepare a simulation table using Time Advanced Algorithm:

IAT : 1 6 1 3 5 7 1 4

ST : 4 2 5 5 1 4 4 4 1

i) Compute Departure Time

C. No	IAT	AT	ST	DT
1	–	0	4	4
2	1	1	2	6
3	6	7	5	12
4	1	8	5	17
5	3	11	1	18
6	5	16	4	22
7	7	23	4	27
8	1	24	4	31
9	4	28	1	32

6. Prepare a simulation table using Time advanced algorithm with IAT : 1 1 8 6 8

(4)

ST : 4 2 3 4 1 2

Find customers who spent more than 4 min.

i) Compute Departure Time

C. No	IAT	AT	ST	DT
1	–	0	4	4
2	1	1	2	6
3	1	2	3	9
4	8	10	4	14
5	6	16	1	17
6	8	24	2	26

ii) Simulation table of 6 customers.

Event type	Clock	S/m state		Check out time	Future Event List	CS				
		LQ(t)	LS(t)			S	N_D	F	B	MQ
A_1	0	0	1	$(C_1, 0)$	(A, 1) (D, 4)	0	0	0	0	0
A_2	1	1	1	$(C_1, 0)$ $(C_2, 1)$	(A, 2) (D, 4)	0	0	0	1	1
A_3	2	2	1	$(C_1, 0)$ $(C_2, 1)$ $(C_3, 2)$	(D, 4) (A, 10)	0	0	0	2	2
D_1	4	1	1	$(C_2, 1)$ $(C_3, 2)$	(D, 6) (A, 10)	4	1	0	4	2
D_2	6	0	1	$(C_3, 2)$	(D, 9) (A, 10)	9	2	1	6	2
D_3	9	0	0	–	(A, 10) (D, 14)	16	3	2	9	2
A_4	10	0	1	$(C_4, 10)$	(D, 14) (A, 16)	16	3	2	9	2
D_4	14	0	0	–	(A, 16) (D, 17)	20	4	2	13	2
A_5	16	0	1	$(C_5, 16)$	(D, 17) (A, 24)	20	4	2	13	2
D_5	17	0	0	–	(A, 24) (D, 26)	21	5	2	14	2
A_6	24	0	1	$(C_6, 24)$	(D, 26)	21	5	2	14	2
D_6	26	0	0	–	–	23	6	2	16	2

7. Consider single server queue with one checkout counter using ES/TA algorithm

IAT : 4 2 8 1 8 3 6 8

ST : 4 6 5 2 3 4 4 1

Find the no. of customers who spent 4 or more ~~min~~ in the system. Stopping time = 32

i) Compute Arrival & Departure time

C.No	IAT	AT	ST	DT
1	–	0	4	4
2	4	4	6	10
3	2	6	5	15
4	8	14	2	17
5	1	15	3	20
6	8	23	4	27
7	3	26	4	31
8	6	32	1	33
9	8	40	–	–

ii) Simulation table

Event type	clock	S/m state LQ(t)	S/m state LS(t)	check out time	Future Event List	CS: S	N_D	F	B	MQ
A_1	0	0	1	$(C_1,1)$	(A,4) (D,4) (E,32)	0	0	0	0	0
D_1/A_2	4	0	1	$(C_2,4)$	(A,6) (D,10) (E,32)	4	1	1	4	0
A_3	6	1	1	$(C_2,4)$ $(C_3,6)$	(D,10) (A,14) (E,32)	4	1	1	6	1
D_2	10	0	1	$(C_3,6)$	(A,14) (D,15) (E,32)	10	2	2	10	1
A_4	14	1	1	$(C_3,6)$ $(C_4,14)$	(A,15) (D,15) (E,32)	10	2	2	14	1
A_5/D_3	15	1	1	$(C_4,14)$ $(C_5,15)$	(D,17) (A,23) (E,32)	19	3	3	15	1
D_4	17	0	1	$(C_5,15)$	(D,20) (A,23) (E,32)	22	4	3	17	1
D_5	20	0	0	-	(A,23) (D,27) (E,32)	27	5	4	20	1
A_6	23	0	1	$(C_6,23)$	(A,26) (D,27) (E,32)	27	5	4	20	1

(5)	D_6	27	0	1	$(C_7, 26)$	(D,31) (A,32) (E,32)	31	6	5	24	1
	D_7	31	0	0	–	(A,32) (D,33) (E,32)	36	7	6	28	1
	A_8	32	0	1	$(C_8, 32)$	(A,40) (D,33) (E,32)	36	7	6	28	1

8. Develop a simulation table for single server queue with one check out counter using TA algorithm. Find busy time of server, maximum queue length, Total no. of customer who spent 3 min or more in system, Total number of departure.

IAT : 1 6 8 8 3 8 4 2 8

ST : 4 1 4 4 2 3 5 6 4

i) Compute Arrival and Departure time

C.No	IAT	AT	ST	DT
1	–	0	4	4
2	1	1	1	5
3	6	7	4	11
4	8	15	4	19
5	8	23	2	25
6	3	26	3	29
7	8	34	5	39
8	4	38	6	45
9	2	40	4	49
10	8	48	–	–

Event type	Clock	S/m state LQ(t)	S/m state LS(t)	Check Out Time	Future Event List	CS S	CS N_D	CS F	CS B	CS MQ
A_1	0	0	1	$(C_1, 0)$	(A, 1) (D, 4)	0	0	0	0	0
A_2	1	1	1	$(C_1, 0)(C_2, 1)$	(A, 7) (D, 4)	0	0	0	1	1
D_1	4	0	1	$(C_2, 1)$	(A, 7) (D, 5)	4	1	1	4	1
D_2	5	0	0	–	(A, 7) (D, 11)	8	2	2	5	1
A_3	7	0	1	$(C_3, 7)$	(A, 15) (D, 11)	8	2	2	5	1
D_3	11	0	0	–	(A, 15) (D, 19)	12	3	3	9	1
A_4	15	0	1	$(C_4, 15)$	(D, 19) (A, 23)	12	3	3	9	1
D_4	19	0	0	–	(A, 23) (D, 25)	16	4	4	13	1
A_5	23	0	1	$(C_5, 23)$	(D, 25) (A, 26)	16	4	4	13	1
D_5	25	0	0	–	(A, 26) (D, 29)	18	5	4	15	1
A_6	26	0	1	$(C_6, 26)$	(D, 29) (A, 34)	18	5	4	15	1
D_6	29	0	0	–	(A, 34) (D, 39)	21	6	5	18	1
A_7	34	0	1	$(C_7, 34)$	(A, 38) (D, 39)	21	6	5	18	1
A_8	38	1	1	$(C_7, 34)(C_8, 38)$	(D, 39) (A, 40)	21	6	5	22	1
D_7	39	0	1	$(C_8, 38)$	(A, 40) (D, 45)	26	7	6	23	1
A_9	40	1	1	$(C_8, 38)(C_9, 40)$	~~(A, 48)~~ (D, 45)	26	7	6	24	1
D_8	45	0	1	$(C_9, 40)$	~~(A, 48)~~ (D, 49)	33	8	7	29	1
[illegible]	[illegible]	[illegible]	[illegible]	~~$(C_9, 40)(C_{10}, 48)$~~	~~(D, 49)~~	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
D_9	49	0	0	$(C_{10}, 48)$	–	42	9	8	33	1

Busy time of server = 33 min

Maximum Queue Length = 1

Total no. of customer who spent 3 or more in System = 8

Total no. of departure = 9

Travel Times as given below: Consider stopping time 32 clock cycle.
(OR) The stopping event will be completion of 2 weighings (2 AQw).

Loading Times	5	15	10	15	15	10	5
Weighing Times	17	15	15	15	15	12	
Travel Times	40	60	60	60	80		

Solution:

Simulation table for Dump-truck Operation.

Clock t	System State				Lists		Future Event List	Cumulative statistics	
	LQ(t)	L(t)	WQ(t)	W(t)	Loader Queue	Weigh Queue		B_L	B_S
0	4	2	0	1	DT_4 DT_5 DT_6 DT_7	–	$(E_L, 5, DT_2)$* $(E_L, 15, DT_3)$ $(E_W, 17, DT_1)$	0	0
5	3	2	1	1	DT_5 DT_6 DT_7	DT_2	$(E_L, 15, DT_3)$* $(E_W, 17, DT_1)$ $(E_L, 15_{(5+10)}, DT_4)$	10 $(5-0)*2+0$	5 $(5-0)+0$
15	2	2	2	1	DT_6 DT_7	DT_2 DT_3	$(E_W, 17, DT_1)$ $(E_L, 15, DT_4)$* $(E_L, 30_{(15+15)}, DT_5)$	30 $(15-5)*2+10$	15 $(15-5)+5$
15	1	2	3	1	DT_7	DT_2 DT_3 DT_4	$(E_W, 17, DT_1)$* $(E_L, 30, DT_5)$ $(E_L, 30, DT_6)$	30	15
17	1	2	2	1	DT_7	DT_3 DT_4	$(E_L, 30, DT_5)$* $(E_L, 30, DT_6)$ $(A_Q, 57, DT_1)$ (17+40) $(E_W, 32, DT_2)$ (17+15)	34	17
30	0	2	3	1	–	DT_3 DT_4 DT_5	$(E_L, 30, DT_6)$* $(A_Q, 57, DT_1)$ $(E_W, 32, DT_2)$ $(E_L, 40, DT_7)$	60	30
30	0	1	4	1	–	DT_3 DT_4 DT_5 DT_6	$(A_{QL}, 57, DT_1)$ $(E_W, 32, DT_2)$* $(E_L, 40, DT_7)$	60	30
32	0	1	3	1	–	DT_4 DT_5 DT_6	$(A_{QL}, 57, DT_1)$ $(E_L, 40, DT_7)$ $(A_{QL}, 92, DT_2)$ $(E_W, 47, DT_3)$	62	32

2. Consider 6 Dump-trucks with loading times, weighing time & Traveling times are given below,

Loading Times	5	5	10	15	20	5	5
Weighing Times	12	15	20	12	15	15	
Travel Time	40	60	20	80			

- Until clock cycle 52
- Calculate
  i) Avg loader utilization
  ii) Avg scale utilization.

Solution:

Simulation table for Dump truck operations.

Clock t	System State LQ(t)	L(t)	WQ(t)	W(t)	Lists: Loader queue	Weigh queue	Future Event List	Cumulative Statistics B_L	B_S
0	3	2	0	1	DT_4 DT_5 DT_6	–	$(E_L, 5, DT_2)$* $(E_L, 5, DT_3)$ $(E_W, 12, DT_1)$	0	0
5	2	2	1	1	DT_5 DT_6	DT_2	$(E_L, 5, DT_3)$* $(E_W, 12, DT_1)$ $(E_L, 15, DT_4)$	10	5
5	1	2	2	1	DT_6	DT_2 DT_3	$(E_W, 12, DT_1)$* $(E_L, 15, DT_4)$ $(E_L, 20, DT_5)$	10	5
12	1	2	1	1	DT_6	DT_3	$(E_L, 15, DT_4)$* $(E_L, 20, DT_5)$ $(AQ_L, 52, DT_1)$ $(E_W, 27, DT_2)$	24	12
15	0	2	2	1	–	DT_3 DT_4	$(E_L, 20, DT_5)$* $(AQ_L, 52, DT_1)$ $(E_W, 27, DT_2)$ $(E_L, 35, DT_6)$	30	15
20	0	1	3	1	–	DT_3 DT_4 DT_5	$(AQ_L, 52, DT_1)$ $(E_W, 27, DT_2)$* $(E_L, 35, DT_6)$	40	20
27	0	1	2	1	–	DT_4 DT_5	$(AQ_L, 52, DT_1)$ $(E_L, 35, DT_6)$* $(AQ_L, 87, DT_2)$ $(E_W, 47, DT_3)$	47	27
35	0	0	3	1	–	DT_4 DT_5 DT_6	$(AQ_L, 52, DT_1)$ $(AQ_L, 87, DT_2)$ $(E_W, 47, DT_3)$*	55	35

Clock t	System State				Lists		Future event List	Cumulative statistics	
	LQ(t)	L(t)	WQ(t)	W(t)	Loader queue	weigh queue		B_L	B_S
47	0	0	2	1	–	DT_5 DT_6	(ALQ, 52, DT_1) * (ALQ, 87, DT_2) (ALQ, 67, DT_3) (E_W, 59, DT_4)	55	47
52	0	1	2	1	–	DT_5 DT_6	(ALQ, 87, DT_2) (ALQ, 67, DT_3) (E_L, 57, DT_1) [52+5] (E_W, 59, DT_4)	55	52

i) Average loader utilization $= \frac{55/2}{52} = \underline{0.529}$

ii) Average scale utilization $= \frac{52}{52} = \underline{1.00}$

3. Consider Loading Times, weighing Times & Travel times of 6 Dump trucks. Loading Times & weighing Times are based on FIFO, Until clock cycle 60 [or] entire system runs for 1 hours [or] estimation time is 60 mins.

Loading Times	10	10	15	5	10	5
Weighing Times	10	15	5	10	10	
Travel Times	40	60	80	100		

* if all the trucks have equal value use FIFO method.

Solution:

Simulation table for Dump-truck operation.

Clock t	System State				Lists		Future event List	Cumulative Statistics	
	LQ(t)	L(t)	WQ(t)	W(t)	Loaders queue	weigh queue		B_L	B_S
0	3	2	0	1	DT_4 DT_5 DT_6	–	(E_L, 10, DT_2) (E_L, 10, DT_3) (E_W, 10, DT_1)*	0	0
10	2	2	0	1	DT_5 DT_6	–	(E_L, 10, DT_3)* (ALQ, 50, DT_1) [10+40] (E_W, 25, DT_2) [10+15] (E_L, 25, DT_4) [10+15]	20	10
10	1	2	1	1	DT_6	DT_3	(ALQ, 50, DT_1) (E_W, 25, DT_2) (E_L, 25, DT_4) (E_L, 15, DT_5)* [10+5]	20	10

Clock t	System State				Lists		Future Event List	Cumulative Statistics	
	LQ(t)	L(t)	WQ(t)	W(t)	Loader Queue	Weigh Queue		B_L	B_S
15	0	2	2	1	–	DT_3 DT_5	$(A_{QL}, 50, DT_1)$ $(E_W, 25, DT_2)$* $(E_L, 25, DT_4)$ $(E_L, 25, DT_6)$	30	15
25	0	2	1	1	–	DT_5	$(A_{QL}, 50, DT_1)$ $(E_L, 25, DT_4)$* $(E_L, 25, DT_6)$ $(A_{QL}, 85, DT_2)$ $(E_W, 30, DT_3)$	50	25
25	0	1	2	1	–	DT_5 DT_4	$(A_{QL}, 50, DT_1)$ $(E_L, 25, DT_6)$* $(A_{QL}, 85, DT_2)$ $(E_W, 30, DT_3)$	50	25
25	0	0	3	1	–	DT_5 DT_4 DT_6	$(A_{QL}, 50, DT_1)$ $(A_{QL}, 85, DT_2)$ $(E_W, 30, DT_3)$*	50	25
30	0	0	2	1	–	DT_4 DT_6	$(A_{QL}, 50, DT_1)$ $(A_{QL}, 85, DT_2)$ $(A_{QL}, 110, DT_3)$ $(E_W, 40, DT_5)$*	50	30
40	0	0	1	1	–	DT_6	$(A_{QL}, 50, DT_1)$* $(A_{QL}, 85, DT_2)$ $(A_{QL}, 110, DT_3)$ $(A_{QL}, 140, DT_5)$ $(E_W, 50, DT_4)$	50	40
50	0	1	1	1	–	DT_6	$(A_{QL}, 85, DT_2)$ $(A_{QL}, 110, DT_3)$ $(E_L, 55, DT_1)$ $(E_W, 50, DT_4)$* $(A_{QL}, 140, DT_5)$	50	50
50	0	1	0	1	–	–	$(A_{QL}, 85, DT_2)$ $(A_{QL}, 110, DT_3)$ $(A_{QL}, 140, DT_5)$ $(A_{QL}, 90, DT_4)$ $(E_W, 60, DT_6)$ $(E_L, 55, DT_1)$*	50	50
55	0	0	1	1	–	DT_1	$(A_{QL}, 85, DT_2)$ $(A_{QL}, 110, DT_3)$ $(A_{QL}, 140, DT_5)$ $(A_{QL}, 90, DT_4)$ $(E_W, 60, DT_6)$*	55	55

Clock	System State				Lists		Future Event List	Cumulative Statistics	
t	$LQ(t)$	$L(t)$	$WQ(t)$	$W(t)$	Loader Queue	Weigh queue		B_L	B_S
60	0	0	0	1	—	—	$(ALQ, 85, DT_2)$ $(ALQ, 110, DT_3)$ $(ALQ, 140, DT_2)$ $(ALQ, 90, DT_4)$ $(ALQ, 120, DT_6)$ $(EW, 70, DT_1)$	55	60