

unit 1:Introduction to simulation

1. Simulation:

- **Simulation is the imitation of the operation of a real world process or system over time.**
- **Simulation models help us to study the behavior of system as it evolves**
- **models keeps the set of assumption concerning the operation of the system**
- **Assumptions are expressed in terms of mathematical, logical and symbolic relationship between the entities or object of interest of the system.**
- **Simulation modeling can be used both as an analysis tools to predict the performance of the new system and also predict the effect of changes to existing system.**
- **simulation can be done by hand or computer its keeps the history of system**
- **Simulation produce the set of data is used to estimate the measures of performance of system.**

1.1 When Simulation is the Appropriate Tool:

- **Study of and experimentation** with the internal interactions of a complex system, or of a subsystem within a complex system.
- Informational, organizational and environmental changes can be simulated and **the model's behavior can be observer.**
- The knowledge gained in designing a simulation model can be of great **value toward suggesting improvement in the system under investigation.**
- By changing simulation inputs and observing the resulting outputs, valuable insight may be obtained into which variables are most important and how variables interact.
- Simulation can be used as a **pedagogical (teaching) device** to reinforce analytic solution methodologies.
- Can be used to experiment **with new designs or policies prior to implementation**, so as to prepare for what may happen.
- Can be used to **verify analytic solutions.**
- By simulating different capabilities for a machine, requirements can be determined.
- Simulation models **designed for training, allow learning without the cost and disruption of on-the-job instructions.**
- **Animation** shows a system in simulated operation so that **the plan can be visualized.**
- **The modern system (factory, water fabrication plant, service organization, etc) is so complex** that the interactions can be treated only through simulation

1.2 When Simulation is Not Appropriate

- Ñ Simulation should **not be used when the problem can be solved using common sense.**
- Ñ Simulation should **not be used** if the problem can be **solved analytically.**
- Ñ Simulation should **not be used** if it is easier to perform **direct experiments.**
- Ñ Simulation should **not be used**, if the **costs exceeds** savings.
- Ñ Simulation should **not be used** if the **resources or time are not available.**
- Ñ No data is available, not even estimate simulation is not advised.

- Ñ If there is not enough time or the people are not available, simulation is not appropriate.
- Ñ If managers have unreasonable expectation say, too much soon – or the power of simulation is over estimated, simulation may not be appropriate.
- Ñ **If system behavior is too complex or cannot be defined**, simulation is not appropriate

1.3 Advantages of Simulation

1. **New policies, operating procedures, decision rules, information flow, etc can be explored** without disrupting the ongoing operations of the real system.
2. **New hardware designs, physical layouts, transportation systems** can be tested without committing resources for their acquisition.
3. **Hypotheses** about how or why certain phenomena occur can be **tested for feasibility**.
4. **Time can be compressed or expanded allowing for a speedup or slowdown** of the phenomena under investigation.
5. Insight can be obtained about the **interaction of variables**.
6. Insight can be obtained about **the importance of variables to the performance of the system**.
7. **Bottleneck analysis** can be performed indication where work-in process, information materials and so on are being excessively delayed.
8. A **simulation study can help in understanding how the system operates** rather than how individuals think the system operates.
9. “what-if” questions can be answered. **Useful in the design of new systems**.

1.4 Disadvantages of simulation

1. Model building **requires special training**. It is an art **that is learned over time and through experience**.
2. If two models are **constructed by two competent individuals**, they may have similarities, but it is highly unlikely that they will be the same.
3. Simulation results may be **difficult to interpret**. Since most simulation outputs are essentially random variables (they are usually based on random inputs), it may be hard to determine whether an observation is a result of system interrelationships or randomness.
4. Simulation modeling and analysis can be **time consuming and expensive**. Skimping on resources for modeling and analysis may result in a simulation model or analysis that is not sufficient for the task.
5. Simulation is used in some cases when an analytical solution is possible, or even preferable. This might be particularly true in the simulation of some waiting lines where closed-form queueing models are available.

1.5 Applications of Simulation

- **Manufacturing application**
- Semiconductor manufacturing
- construction engineering
- military application
- Business process simulation
- Human system

1. Manufacturing Applications

- Analysis of electronics assembly operations
- Design and evaluation of a selective assembly station for high-precision scroll compressor shells
- Comparison of dispatching rules for semiconductor manufacturing using large-facility models
- Evaluation of cluster tool throughput for thin-film head production
- Determining optimal lot size for a semiconductor back-end factory
- Optimization of cycle time and utilization in semiconductor test manufacturing
- Analysis of storage and retrieval strategies in a warehouse
- Investigation of dynamics in a service-oriented supply chain
- Model for an Army chemical munitions disposal facility

2. Semiconductor Manufacturing

- Comparison of dispatching rules using large-facility models
- The corrupting influence of variability
- A new lot-release rule for wafer fabs
- Assessment of potential gains in productivity due to proactive reticle management
- Comparison of a 200-mm and 300-mm X-ray lithography cell
- Capacity planning with time constraints between operations
- 300-mm logistic system risk reduction

3. Construction Engineering

- Construction of a dam embankment
- Trenchless renewal of underground urban infrastructures
- Activity scheduling in a dynamic, multi project setting
- Investigation of the structural steel erection process
- Special-purpose template for utility tunnel construction

4. Military Application

- Modeling leadership effects and recruit type in an Army recruiting station
- Design and test of an intelligent controller for autonomous underwater vehicles
- Modeling military requirements for non war fighting operations
 - Using adaptive agent in U.S Air Force pilot retention

5. Logistics, Transportation, and Distribution Applications

- Evaluating the potential benefits of a rail-traffic planning algorithm
- Evaluating strategies to improve railroad performance
- Parametric modeling in rail-capacity planning
- Analysis of passenger flows in an airport terminal
- Proactive flight-schedule evaluation
- Logistics issues in autonomous food production systems for extended-duration space exploration
- Sizing industrial rail-car fleets
- Product distribution in the newspaper industry
- Design of a toll plaza

- Choosing between rental-car locations
- Quick-response replenishment

6. Business Process Simulation

- Impact of connection bank redesign on airport gate assignment
- Product development program planning
- Reconciliation of business and systems modeling
- Personnel forecasting and strategic workforce planning

7. Human Systems and Healthcare

- Modeling human performance in complex systems
- Studying the human element in air traffic control
- Modeling front office and patient care in ambulatory health care practices
- Evaluating hospital operations b/n the emergency department and a medical
- Estimating maximum capacity in an emergency room and reducing length of stay in that room.

1.6 Systems and System Environment

System:

System is defined as a group of object that are joined together in some regular interaction or interdependence toward the accomplishment of same.

System environment:

A system is often affected by changes occurring outside the system,Such changes are said to occure in the system environment.

1.7 Components of a System

- 1) **Entity:** An entity is an object of interest in a system.
Ex: In the factory system, departments, orders, parts and products are the entities.
- 2) **Attribute:** An attribute denotes the property of an entity.
Ex: Quantities for each order, type of part, or number of machines in a department are attributes of factory system.
- 3) **Activity:** Represent a time period of specified length
Ex: Manufacturing process of the department.
- 4) **State of the System:** The state of a system is defined as the collection of variables necessary to describe a system at any time, relative to the objective of study.
- 5) **Event:** An event is defined as an instantaneous occurrence that may change the state of the system.

Endogenous : IS used to descried activites and events occurring with in the system

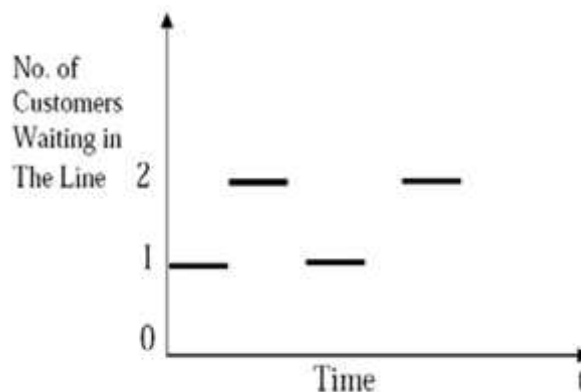
Exogenous: Is used to descried activites and events in the environment that affect the system.

Examples of system and components					
System	Entities	Attributes	Activities	Events	State variables
Banking	Customers	Checking-account balance	Making deposits	Arrival; departure	No. of busy tellers; no. of customers waiting.
Rapid rail	Riders	Origination; destination	Traveling	Arrival at station; arrival at destination	No. of riders waiting at each station; No. of riders in transit
Production	Machines	Speed; capacity; breakdown rate length	Welding; stamping	Breakdown	Status of machines (busy, idle or down)
Inventory	Warehouse	Capacity	Withdrawing	Demand	Levels of inventory; backlogged demands

1.8 Discrete and Continuous Systems

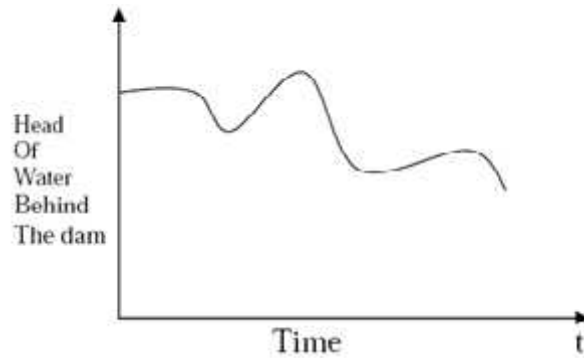
Discrete System:

- Is one in which the state variable change only at a discrete set of points in time.
- The bank is an example, since the state variable the number of customer in the bank changes only when a customer arrives or when the service provided a customer is completed.



Continuous system:

- Is one in which the state variable change continuous over time.
- head of water behind a dam, during and for some time after a rain storm water flow into the lake behind the dam.



1.9 Model of a system

- A **model** is defined as a representation of a system for the purpose of studying the system.
- It is necessary to consider only those aspects of the system that affect the problem under investigation.
- These aspects are represented in a model, and by definition it is a simplification of the system.

Types of Models:

- **Mathematical or physical model**
- **Static and dynamic model**
- **deterministic and stochastic model**
- **discrete and continuous model**

1.Mathematical or physical model:

Mathematical model uses symbolic notation and equations to represents a system

2.Static model:

A static simulation models represent a system at a particular point in time it is also called as monte carlo simulation.

3.dynamic model:

A dynamic simulation models represent system as the change over time. simulation of a bank from 9 to 4 is an example

4.Deterministic model:

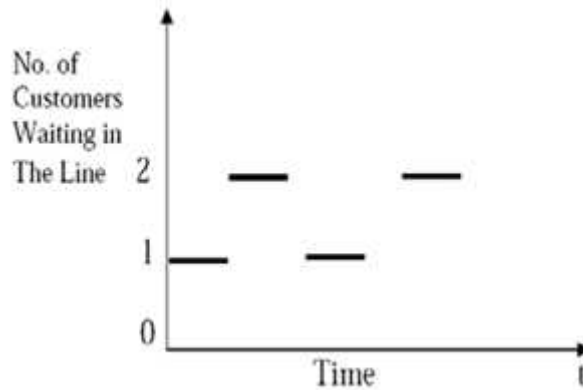
A simulation variable that contain no random variable, have a set of known input which will result in a unique set of output.

5.Stochastic model:

A stochastic simulation model has one or more random **variable** as input. Random input lead to random output.Since the output are random they can be consider only as estimates of the true characteristics of a model.

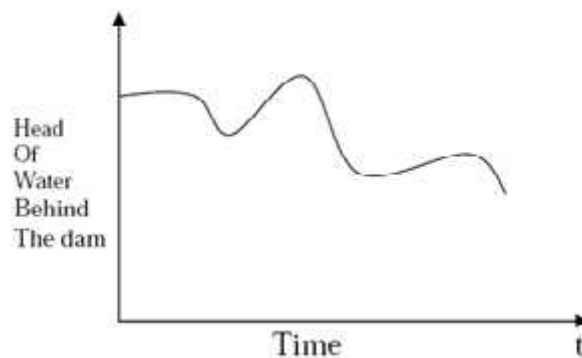
6.Discrete System:

- Is one in which the state variable change only at a discrete set of points in time.
- The bank is an example, since the state variable the number of customer in the bank changes only when a customer arrives or when the service provided a customer is completed.



7. Continuous system:

- Is one in which the state variable change continuous over time.
- head of water behind a dam, during and for some time after a rain storm water flow into the lake behind the dam.



1.10 Discrete event system simulation:

- The model of system in which state variable changes only at a discrete set of points in times
- The simulation models are analyzed by numerical rather than by analytical methods.
- Analytical methods employ the deductive reasoning of mathematics to solve the model. E.g.: Differential calculus can be used to determine the minimum cost policy for some inventory models.
- Numerical methods use computational procedures and are 'runs', which is generated based on the model assumptions and observations are collected to be analyzed and to estimate the true system performance measures.
- Real-world simulation is so vast, whose runs are conducted with the help of computer. Much insight can be obtained by simulation manually which is applicable for small

systems.

1.11 Steps in a simulation study:

1. Problem formulation
2. Setting of objectives and overall project plan
3. model conceptualization
4. data Collection
5. model translation
6. verified
7. validated
8. Experimental design
9. production runs and analysis
10. more runs
11. documentation and reporting
12. Implementation

1. Problem formulation:

- Every study should begin with a statement of the problem.
- If the statement is provided by the policy makers or those that have the problem, The analyst must ensure that the problem being described is clearly understood
- If the problem statement is being developed by the analyst, it is important that the policy makers understand and agree with the formulation.

2. Setting of objective and overall project plan:

- The objectives indicate the questions to be answered by simulation.
- At this point a determination should be made concerning whether simulation is the appropriate methodology. Assuming that it is appropriate,
- the overall project plan should include the study in terms of
 - A statement of the alternative systems
 - A method for evaluating the effectiveness of these alternatives
 - Plans for the study in terms of the number of people involved
 - Cost of the study
 - The number of days required to accomplish each phase of the work with the anticipated results.

3. Model Conceptualization:

- The construction of a model of a system is probably as much art as science.
- The art of modeling is enhanced by ability to have following:
 - To abstract the essential features of a problem.
 - To select and modify basic assumptions that characterizes the system.
 - To enrich and elaborate the model until a useful approximation results.

4. Data Collection:

- There is a constant interplay between the construction of the model and the collection of the needed input data.
- As complexity of the model changes the required data elements may also change.
- Since data collection takes such a large portion of the total time required to perform a simulation it is necessary to begin it as early as possible.

5. Model Translation:

- Since most real world system result in model that require a great deal of information storage and computation, the model must be entered into a computer recognizable format.
- we use term program even though it is possible to accomplish the desired result in many instances with little or no actual coding.

6. Varified:

- It pertains to the computer program and checking the performance.
- If the input parameters and logical structure and correctly represented, verification is completed.

7. Validated:

- validation is the determination that a model is an accurate representation of the real system.
- Is usually achieved through the calibration of the model an iterative process of comparing the model to actual system behavior and using the discrepancy between the two and the insights gained to improve the model.
- This process is repeated until model accuracy is judges acceptable.

8. Experimental Design:

- The alternatives that are to be simulated must be determined. For each system design, decisions need to be made concerning
 - a. Length of the initialization period
 - b. Length of simulation runs
 - c. Number of replication to be made of each run

9. Production runs and analysis:

- They are used to estimate measures of performance for the system designs that are being simulated.

10. More runs:

- Based on the analysis of runs that have been completed. The analyst determines if additional runs are needed and what design those additional experiments should follow.

11.Documentation and reporting:

Two types of documentation. Program documentation and Process documentation

- Ñ **Program documentation:** Can be used again by the same or different analysts to understand how the program operates
- Ñ **Process documentation:** This enable to review the final formulation and alternatives, results of the experiments and the recommended solution to the problem. The final report provides a vehicle of certification.

12.Implementation:

Success depends on the previous steps. If the model user has been thoroughly involved and understands the nature of the model and its outputs, likelihood of a vigorous implementation is enhanced.



1.12 Simulation of queuing systems

A **Queuing system** is described by its **calling population**, the **nature of its arrivals**, the **service mechanism**, the **system capacity**, and **queuing discipline**.

Simulation is often used in the analysis of queuing models. In a simple typical queuing model, shown in

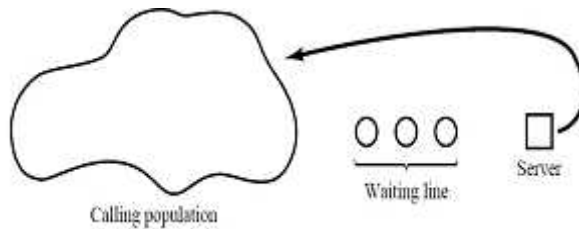


Figure 1. Queuing system.

- In the single-channel queue, the calling population is **infinite**; that is, if a **unit leaves the calling population and joins the waiting line or enters service, there is no change in the arrival rate of other units that may need service**.
- **Arrivals for service occur one at a time in a random fashion**; once they **join the waiting line**, they are eventually served.
- The **system capacity has no limit**, meaning that any number of units can wait in line. Finally, **units are served in the order of their arrival (often called FIFO: first in, first out) by a single server or channel**.
- **Arrivals and services** are defined by the **distributions of the time between arrivals** and the **distribution of service times**, respectively.
- The **state of the system**: the number of units in the system and the status of the server, busy or idle.
- **An event**: a set of circumstances that cause an instantaneous change in the state of the system. In a single-channel queuing system there are only two possible events that can affect the state of the system.
- The **simulation clock** is used to track simulated time.

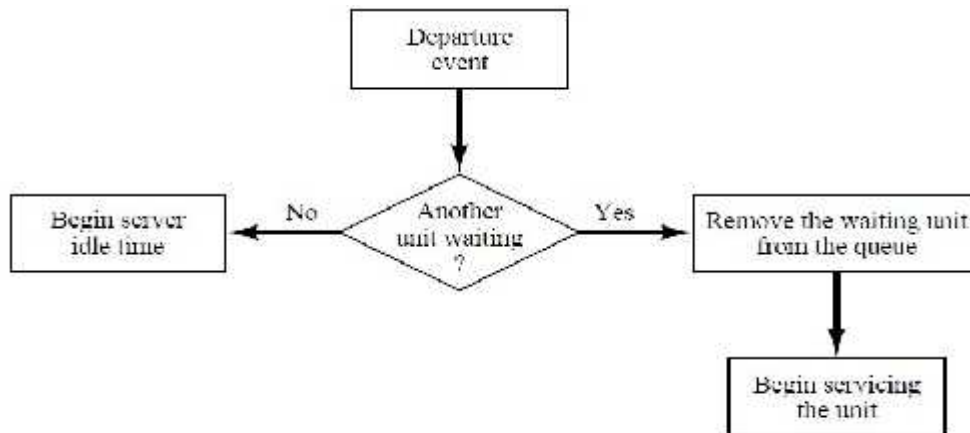


Figure 2.2 Service-just-completed flow diagram.

- The arrival event occurs when a unit enters the system. **The flow diagram for the arrival event is shown in**

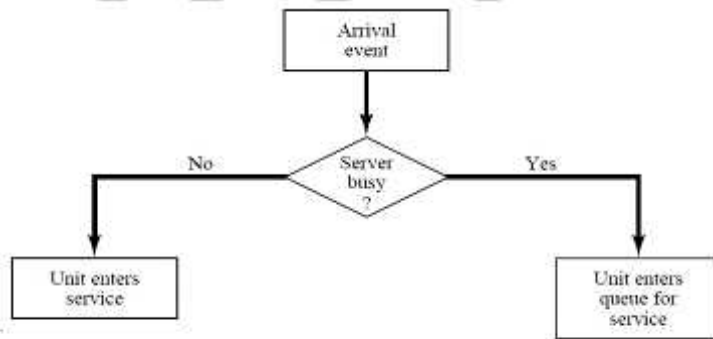


Figure Unit-entering-system flow diagram.

- The unit may find the **server either idle or busy**; therefore, either the unit begins service immediately, or it enters the queue for the server. The unit follows the course of action shown in fig 2.4.
- If the server is busy, the unit enters the queue. If the server is idle and the queue is empty, the unit begins service. It is not possible for the server to be idle and the queue to be nonempty.

		Queue status	
		Not empty	Empty
Server status	Busy	Enter queue	Enter queue
	Idle	Impossible	Enter service

Figure 2.4 Potential unit actions upon arrival.

- After the completion of a service the service may become idle or remain busy with the next unit. The relationship of these two outcomes to the status of the queue is shown in fig 2.5. If the queue is not empty, another unit will enter the server and it will be busy

		Queue status	
		Not empty	Empty
Server outcomes	Busy	/	Impossible
	Idle	Impossible	/

Figure 2.5 Server outcomes after service completion.

Problems:

Single channel queuing system problem formulas:

1. Time Customer wait in queue= Time service begin – Arrival Time
2. Time Service End= Service time + Time service begin
3. Time customer Spend In system= Time service end-Arrival Time
4. Idel Time of Server=Time service Begin(N)-Time Service end(N-1)

Standard Formulas:

1.**Average waiting time(i.e customer wait)**=total time customer wait in queue / Total number of customer

2.**Probability(Wait i.e customer wait)**=Number of Customer who wait / Total number of customer

3.**Probability of idle server(idle time of server)**=total idle time of server / total run time of simulation

4.**average service time**=total service time/total number of customer

5.**average times between arrivals**=sum of all times between arrival/number of arrivals-1

6.**Average waiting time those who wait in queue**=total time customer wait in queue/total number of customer who wait

7.**Average time customer spend In the system**=Total time customer spend in system/total number of customer

① Single channel and Multi channel (Able Baker prob.)
Queuing system problem : UNIT-1

$$\text{Average Waiting time (of customer)} = \frac{\text{Total time customer waiting in queue}}{\text{Total no. of customers}}$$

$$\text{Probability wait (of customer)} = \frac{\text{No. of customers to wait}}{\text{Total no. of customers}}$$

$$\text{Probability of idle server (Idle time of server)} = \frac{\text{Total idle time of server}}{\text{Total run time of simulation (TSE last value)}}$$

$$\text{Average Service Time} = \frac{\text{Total service time}}{\text{Total no. of customers}}$$

$$\text{Average time b/w arrival} = \frac{\text{sum of all time b/w arrival}}{\text{No. of arrival} - 1}$$

$$\text{Average waiting time those who wait in queue} = \frac{\text{Total time customer wait in queue}}{\text{Total no. of customer who wait}}$$

$$\text{Average time customer spends in the system} = \frac{\text{Total time customer spends in S/M}}{\text{Total no. of customers}}$$

AT → Arrival Time

IAT → Inter Arrival Time

ST → Service Time

C.No → Customer Number

RD → Random Digit

RDA → Random Digit Assessment

TSB → Time Service Begin

TSE → Time Service End

CP → Cumulative Probability

P → Probability

1. A small grocery store has only one check out counter. The customer arrives at this check out counter at random from 1 to 8 min apart. Each possible value of service time has same probability of occurrence. The service time varies from 1 to 6 mins apart. Each possible value of service time has same probability of occurrence. Develop simulation distribution table for 8 customers.

Random digit for arrival time :

913 727 015 948 309 922 753 235 302

Service Time : (Random digit)

84 10 74 53 17 79 91 67 89 38

i] Determine Inter Arrival Time distribution table

C.No	$P(1/8)$	Cumulative Probability	Random Digit Assessment
1	0.125	0.125	001-125
2	0.125	0.250	126-250
3	0.125	0.375	251-375
4	0.125	0.500	376-500
5	0.125	0.625	501-625
6	0.125	0.750	626-750
7	0.125	0.875	751-875
8	0.125	1.000	876-000

2) ii) Compute Arrival time from IAT

C.No	Random Digit	IAT	Arrival Time
1	-	-	0
2	913	8	8
3	727	6	14
4	015	1	15
5	948	8	23
6	309	3	26
7	922	8	34
8	753	7	41
9	235	2	43
10	302	3	46

iii) Determine Service Time Distribution Table

S.No	P(1/6)	Cumulative probability	Random Digit Assess
1	0.16	0.16	01-16
2	0.16	0.32	17-32
3	0.16	0.48	33-48
4	0.16	0.64	49-64
5	0.16	0.80	65-80
6	0.16	0.96	81-96

iv) Compute Service Time

S.No	Random digit	Service Time
1	84	6
2	10	1
3	74	5
4	53	4
5	17	2
6	79	5
7	91	6
8	67	5
9	89	6
10	38	3

Y] Simulation Table for 10 customers

C.No	Arrival Time	Service Time	Time Service Begin	Time Service End	TSE - AT	TSB(n) - TSE(n-1)	TSB - AT
1	0	6	0	6	6 (6-0)	0	0
2	8	1	8	9	1 (9-8)	2 (8-6)	0 (8-8)
3	14	5	14	19	5 (19-14)	5 (14-9)	0 (14-14)
4	15	4	19	23	8	0	4 (19-15)
5	23	2	23	25	2	0	0
6	26	5	26	31	5	1	0
7	34	6	34	40	6	3	0
8	41	5	41	46	5	1	0
9	43	6	46	52	9	0	3
10	46	3	52	55	9	0	6

Average waiting Time = $\frac{\text{Total time customer waiting in queue}}{\text{Total no. of customers}} = \frac{4+3+6}{10} = \frac{13}{10} = 1.3 \text{ min}$

Probability of wait = $\frac{\text{No. of customers to wait}}{\text{Total no. of customers}} = \frac{3}{10} = 0.3 \text{ min}$

Probability of idle server = $\frac{\text{Total idle time of server}}{\text{Total run time of simulator}} = \frac{2+5+1+3+1}{55} = \frac{12}{55} = 0.218 \text{ min}$
 (TSE last value)

2. A small grocery store has only one check out counter at random from 1-6 min apart. Each possible value of IAT has the same probability of occurrence. The service time vary from 1 to 6 mins with probability shown below.

ST:	1	2	3	4	5	6
P:	0.10	0.20	0.25	0.30	0.10	0.05

Develop a simulation Table for 10 customers
Take a random digit for arrival;
13 27 15 48 9 22 53 35 2

Average time customer spent in system = $\frac{\text{Total time customer spends in system}}{\text{Total no. of customers}}$

$$= \frac{6+1+5+8+2+5+6+5+9+9}{10} = \frac{56}{10} = 5.6 \text{ min}$$

Average waiting time those who wait in queue = $\frac{\text{Total time customer wait in queue}}{\text{Total no. of customer who wait}}$

$$= \frac{4+3+6}{3} = \frac{13}{3} = 4.3 \text{ min}$$

Average Time b/w arrival = $\frac{\text{Sum of all time b/w arrival}}{\text{No. of arrival} - 1}$

$$= \frac{46}{10-1} = \frac{46}{9} = 5.11 \text{ min}$$

Average Service Time = $\frac{\text{Total Service Time}}{\text{Total no. of customers}}$

$$= \frac{6+1+5+4+2+5+6+5+9+9}{10} = \frac{43}{10} = 4.3 \text{ min}$$

Random digit for Service Time

84 10 74 53 17 91 79 67 38 89 sequentially.

Calc. Avg Service Time, probability of idle Service time, average b/w arrivals & avg. time customer spent in System.

i) Determine IAT Distribution Table

C.No	Probability (1/6)	Cumulative probability	Random digit assessment
1	0.16	0.16	01-16
2	0.16	0.32	17-32
3	0.16	0.48	33-48
4	0.16	0.64	49-64
5	0.16	0.80	65-80
6	0.16	0.96	81-96

ii) Compute Arrival time from IAT

iv) Compute Service Time from Distribution Table

C.No	RD	IAT	AT
1	-	-	0
2	13	1	1
3	27	2	3
4	15	1	4
5	48	3	7
6	9	1	8
7	22	2	10
8	53	4	14
9	35	3	17
10	2	1	18

S.No	RD	ST
1	84	4
2	10	1
3	74	4
4	53	3
5	17	2
6	91	5
7	79	4
8	67	4
9	38	3
10	89	5

iii) Determine ST Dist. Table

S.No	Probability	CP	RDA
1	0.10	0.10	01-10
2	0.20	0.30	11-30
3	0.25	0.55	31-55
4	0.30	0.85	56-85
		0.95	86-95

Simulation Table for 10 customers

o	Arrival Time	Service Time	Time Service Begin	Time Service End	Cust. time spent in s/m	Idle Server Time	Cust. time wait in queue
	0	4	0	4	4	0	0
	1	1	4	5	4	0	3
	3	4	5	9	6	0	2
	4	3	9	12	8	0	5
	7	2	12	14	7	0	5
	8	5	14	19	11	0	6
	10	4	19	23	13	0	9
	14	4	23	27	13	0	9
	17	3	27	30	13	0	10
	18	5	30	35	17	0	12

Average Service Time = $\frac{35}{10} = \underline{\underline{3.5 \text{ min}}}$

probability of Idle Server Time = $\frac{0}{35} = \underline{\underline{0}}$

Average Time b/w arrivals = $\frac{18}{9} = \underline{\underline{2 \text{ min}}}$

Average time Customer spent in system = $\frac{61}{10} = \underline{\underline{6.1 \text{ min}}}$

3. Consider a store with one checkout counter. Prepare simulation table & find out average waiting time of customer in $\&$ waiting queue, probability of idle server, average service time

IAT : 3 2 6 4 4 5 8 7

ST : 4 5 5 8 4 6 2 3 4

Assume 1st customer arrives at $t=0$.

i) Inter arrival distribution Table

C.No	IAT	AT
1	-	0
2	3	3
3	2	5
4	6	11
5	4	15
6	4	19
7	5	24
8	8	32
9	7	39

ii) Simulation table for 9 customers

C.No	AT	ST	TSSB	TSE	Cust. spent in sim.	Idle time of server	Cust. time wait in que
1	0	4	0	4	4	0	0
2	3	5	4	9	6	0	1
3	5	5	9	14	9	0	4
4	11	8	14	22	11	0	3
5	15	4	22	26	11	0	7
6	19	6	26	32	13	0	7
7	24	2	32	34	10	0	8
8	32	3	34	37	5	0	2
9	39	4	39	43	4	2	0

⑤ Average waiting time of customer in waiting queue = $\frac{32}{7} = \underline{\underline{4.57 \text{ min}}}$

Probability of Idle Server = $\frac{2}{10} = \underline{\underline{0.2 \text{ min}}}$

Average Service Time = $\frac{41}{9} = \underline{\underline{4.56 \text{ min}}}$

Multichannel Problems :

Type I

4 Consider a simulation with a restaurant system where car ~~to~~ hope takes order & brings an item to the car. The car arrives in the manner :

Time b/w arrival :	1	2	3	4
Probability :	0.25	0.40	0.20	0.15

Consider 2 persons Able & Baker. Able is better & bit faster than Baker.

Able Service Time :

ST :	2	3	4	5
Prob :	0.30	0.28	0.25	0.17

Baker Service Time :

ST :	3	4	5	6
Prob :	0.35	0.25	0.20	0.20

Take a random digit for arrival :

26 98 90 26 42 74 80 68 22

Random digit for service time :

95 21 51 92 89 38 13 61 50 49

i) Determine Inter Arrival distribution Table

c.No	Probability	Cumulative Probability	Random Digit Assessment
1	0.25	0.25	01-25
2	0.40	0.65	26-65
3	0.20	0.85	66-85
4	0.15	1.00	86-00

ii) Compute AT from IAT distribution Table

c.No	RD	IAT	AT
1	-	-	0
2	26	2	2
3	98	4	6
4	90	4	10
5	26	2	12
6	42	2	14
7	74	3	17
8	80	3	20
9	68	3	23
10	22	1	24

iii) Able Service Time Distribution Table

S.No	Probability	CP	RDA
2	0.30	0.30	01-30
3	0.28	0.58	31-58
4	0.25	0.83	59-83
5	0.17	1.00	84-00

iv) Baker Service Time Distribution Table

S.No	Probability	CP	RDA
3	0.35	0.35	01-35
4	0.25	0.60	36-60
5	0.20	0.80	61-80
6	0.20	1.00	81-00

✓ Simulation table for 10 customers

C.No	AT	RD for service	ST	When Able is Available	When Baker is Available	Server Chosen	Able		Baker		cust. in s/m	Idle Server time	wa. in que
							TSB	TSE	TSB	TSE			
1	0	95	5	0	0	A	0	5	-	-	5	0	0
2	2	21	3	5	0	B	-	-	2	5	3	2	0
3	6	51	3	5	5	A	6	9	-	-	3	1	0
4	10	92	3	9	5	A	10	15	-	-	5	1	0
5	12	89	6	10	5	B	-	-	12	18	6	7	0
6	14	38	3	16	18	A	15	18	-	-	4	0	1
7	17	13	3	18	18	A	18	20	-	-	3	0	1
8	20	61	4	20	18	A	20	24	-	-	4	0	0
9	23	50	4	24	18	B	-	-	23	27	4	5	0
10	24	49	3	24	27	B	24	27	-	-	3	0	0

5. Consider Simulation table for Able & Baker problem where time b/w arrival are :

Time b/w arrival : 1 2 3 4 5

Probability : 0.20 0.15 0.05 0.20 0.40

Baker is faster than Able.

Service Time for Able :

ST : 3 4 5 6 2

Prob : 0.20 0.05 0.15 0.20 0.40

Service Time for Baker :

ST : 2 3 5 6 1

Prob : 0.15 0.20 0.05 0.20 0.40

Random Digit for Arrival :

98 90 42 80 22 26 74 26 68

Random Digit for Service Time :

49 50 61 13 38 89 92 51 21 95

i) Determine IAT distribution Table

C.No	Probability	CP	RDA
1	0.20	0.20	01-20
2	0.15	0.35	21-35
3	0.05	0.40	36-40
4	0.20	0.60	41-60
5	0.40	1.00	61-00

ii) Compute Arrival Time from IAT dist. table.

C.No	RD	IAT	AT
1	-	-	0
2	98	5	5
3	90	5	10
4	42	4	14
5	80	5	19
6	22	2	21
7	26	2	23
8	74	5	28
9	26	2	30
10	68	5	35

iii) Able Service Time Distribution Table

S.No	P	CP	RDA
3	0.20	0.20	01-20
4	0.05	0.25	21-25
5	0.15	0.40	26-40
6	0.20	0.60	41-60
2	0.40	1.00	61-00

iv) Baker Service Time Distribution Table

S.No	P	CP	RDA
2	0.15	0.15	01-15
3	0.20	0.35	16-35
5	0.05	0.40	36-40
6	0.20	0.60	41-60
1	0.40	1.00	61-00

(RD - Random Digit)

v) Simulation table for 10 customers

C.No	Arrival Time	RD of Service	Service Time	When Able is Available	When Baker is Available	Server Chosen	Able		Baker		Cust. time spent in s/m	Idle time of Server	Cust. time wait queue
							TSB	TSE	TSB	TSE			
1	0	49	6	0	0	B	-	-	0	6	6	0	0
2	5	50	6	0	6	A	5	11	-	-	6	5	0
3	10	61	1	11	6	B	-	-	10	11	1	4	0
4	14	13	2	11	11	B	-	-	14	16	2	3	0
5	19	38	5	11	16	B	-	-	19	24	5	3	0
6	21	89	2	11	24	A	21	23	-	-	2	10	0
7	23	92	2	23	24	A	23	25	-	-	2	0	0
8	28	51	6	25	24	B	-	-	28	34	6	4	0
9	30	21	4	25	34	B	30	34	-	-	4	5	0
10	35	95	1	34	34	A	-	-	35	36	1	1	0

Customer time Spent in s/m = TSE - AT

Idle time of Server = TSB(n) - TSE(n-1)

Customer time waiting in queue = TSB - AT

6. Consider Able Baker Dist. table, IAT having equal probability ratio 1 to 6 min apart.

* Able Service Time have equal probability ratio 1-5 min

Baker Service Time have equal probability ratio 1-4 min.

Random Digit for time b/w arrival :

62 89 9 62 24 47 8 86 22

Random Digit for Service Time :

59 12 15 29 28 83 13 16 5 94

Able is faster than Baker.

i) Determine IAT Distribution Table

C.No	P	CP	RDA
1	0.16	0.16	01-16
2	0.16	0.32	17-32
3	0.16	0.48	33-48
4	0.16	0.64	49-64
5	0.16	0.80	65-80
6	0.16	0.96	81-96

ii) Compute Arrival Time from IAT Dist. Table

C.No	RD	IAT	AT
1	-	-	0
2	62	4	4
3	89	6	10
4	9	1	11
5	62	4	15
6	24	2	17
7	47	3	20
8	8	1	21
9	86	6	27
10	22	2	29

- Distribution Table

S.No	P	CP	RDA
1	0.20	0.20	01-20
2	0.20	0.40	21-40
3	0.20	0.60	41-60
4	0.20	0.80	61-80
5	0.20	1.00	81-100

Distribution Table

S.No	P	CP	RDA
1	0.25	0.25	01-25
2	0.25	0.50	26-50
3	0.25	0.75	51-75
4	0.25	1.00	76-100

v) Simulation Table for 10 customers

C.No	AT	RD of Service	ST	When Able is Available	When Baker is Available	Server Chosen	Able		Baker		Cust. spent in s/m	Idle time of Server	Cust. time wait in queue
							TSB	TSE	TSB	TSE			
1	0	59	3	0	0	A	0	3	-	-	3	0	0
2	4	12	1	3	0	A	4	5	-	-	1	1	0
3	10	15	1	5	0	A	10	11	-	-	1	5	0
4	11	29	2	11	0	A	11	13	-	-	2	0	0
5	15	98	15	13	0	A	15	20	-	-	2	2	0
6	17	83	4	20	0	B	-	-	17	21	2	17	0
7	20	13	1	20	21	A	20	21	-	-	1	0	0
8	21	16	1	21	21	A	21	22	-	-	1	0	0
9	27	5	1	22	21	A	27	28	-	-	2	5	0
10	29	94	5	28	21	A	29	34	-	-	5	1	0

7. Develop a simulation table for Abu & Baker problem.

Inter Arrival Times : 3 2 1 5 4 6

Service Time (Abu) : 3 5 2 6 2 1 7

Service Time (Baker) : 2 1 4 5 7 6 2

Abu is much faster than Baker

i) Compute AT from IAT

C. No	IAT	AT
1	-	0
2	3	3
3	2	5
4	1	6
5	5	11
6	4	15
7	6	21

ii) Simulation Table for 7 customers

C.No	AT	ST	When Abu is Available	When Baker is Available	Server chosen	Abu		Baker		Cust. spent in s/m	Idle time of Server	Cust. time wait in queue
						TSS	TSE	TSS	TSE			
1	0	3	0	0	A	0	3	-	-	0	0	3
2	3	5	3	0	A	3	8	-	-	0	0	5
3	5	4	8	0	B	-	-	5	9	0	5	4
4	6	6	8	9	A	8	14	-	-	2	0	8
5	11	7	14	9	B	-	-	11	18	0	2	7
6	15	1	14	18	A	15	16	-	-	0	1	1
7	21	7	16	18	A	21	28	-	-	0	5	7

9) having equal probability ratio 1 to 7 min apart.

Service Time for Able :

ST : 1 2 3 4 5
 Prob : 0.20 0.10 0.30 0.20 0.20

Service Time for Baker :

ST : 1 2 3 4 5
 Prob : 0.10 0.20 0.20 0.30 0.20

Random Visit for Arrival : 95 60 35 40 52 54 10

Random visit for Service : 60 95 35 40 24 54 10 25

Baker is faster than Able.

i) Determine Inter Arrival Distribution Table.

ii) Compute AT from IADT.

C.No	P	CP	RDA
1	0.14	0.14	01-14
2	0.14	0.28	15-28
3	0.14	0.42	29-42
4	0.14	0.56	43-56
5	0.14	0.70	57-70
6	0.14	0.84	71-84
7	0.14	0.98	85-98

C.No	RD	IAT	AT
1	-	-	0
2	95	7	7
3	60	5	12
4	35	3	15
5	40	3	18
6	52	4	22
7	54	4	26
8	10	1	27

iii) Determine Able Service Time Distribution Table

S.No	P	CP	RDA
1	0.20	0.20	01-20
2	0.10	0.30	21-30
3	0.30	0.60	31-60
4	0.20	0.80	61-80
5	0.20	1.00	81-100

iv) Determine Baker Service Time Distribution table

S.No	P	CP	RDA
1	0.10	0.10	1-10
2	0.20	0.30	11-30
3	0.20	0.50	31-50
4	0.30	0.80	51-80
5	0.20	1.00	81-100

v) Simulation Table for 8 customers.

C.No	Arrival Time	RD of Service	Service Time	When Able is Available	When Baker is Available	Server Chosen	Able		Baker		Cust. Spent in \$/m	Idle Server Time	Cust. W in que
							TSB	TSE	TSB	TSE			
1	0	60	4	0	0	B	-	-	0	4	4	0	0
2	7	95	5	0	4	B	-	-	7	12	5	3	0
3	12	35	3	0	12	B	-	-	12	15	3	0	0
4	15	40	3	0	15	B	-	-	15	18	3	0	0
5	18	24	2	0	18	B	-	-	18	20	2	0	0
6	22	54	4	0	20	B	-	-	22	26	4	2	0
7	26	10	1	0	26	B	-	-	26	27	1	0	0
8	27	25	2	0	27	B	-	-	27	29	2	0	0